Table of Contents

KINEMATICS

When working with variables such as these, it is important to make the distinction between scalar and vector quantities:

- Scalar quantities have magnitude (size) only.
- Vector quantities have magnitude as well as direction.

Excluding time, the characteristics of motion are vector quantities, where the direction is equally as important as the magnitude.

- **Displacement** is a vector quantity, which gives the position of a body relative to an origin.
- **Distance** is a scalar quantity, which states how far the body has travelled.
- **Velocity** is a vector quantity, which tells us how fast the body is moving and in what direction.
- **Speed** is a scalar quantity, which tells us how fast the body is moving only. It is the magnitude of the velocity.

STRAIGHT LINE MOTION

s–t GRAPHS

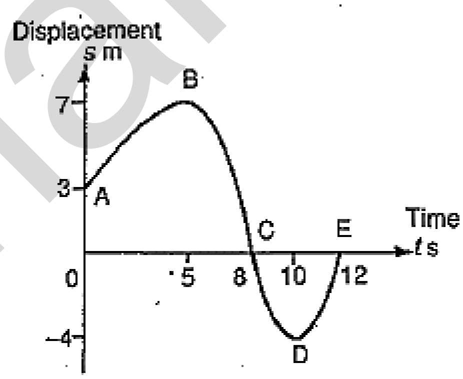


Fig. 20.2

v–t GRAPHS

Another useful graph is the ***v–t* graph,** which relates velocity to time. Fig. 20.3 shows an example of such a graph.

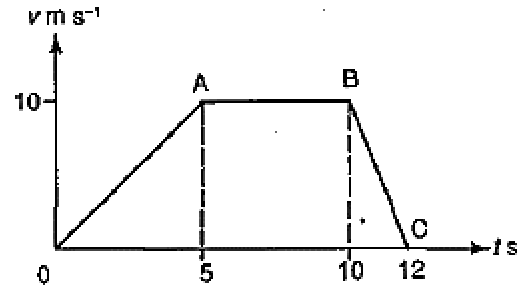


Fig. 20.3

Another useful graph is the v-t graph, which relates velocity to time. Figure 20.3 shows an example of such a graph.

<u>EXAMPLE 1</u>

A train takes 10 minutes to travel between two stations. The train accelerates at a rate of $0.5\,m\,s^{-2}$ for 30 s. It then travels at a constant speed and is finally brought to rest in 15 s with a constant deceleration.

(i) Sketch a velocity–time graph for the journey.

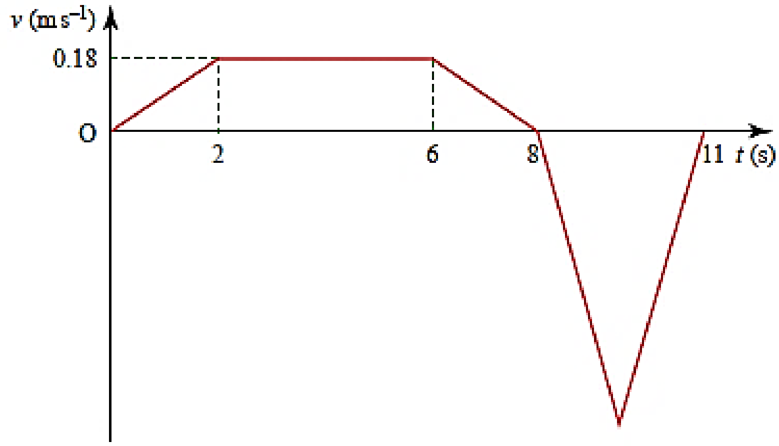
(ii) Find the steady speed, the rate of deceleration and the distance between the two stations.

<u>EXAMPLE 2</u>

On the London underground, Oxford Circus and Piccadilly Circus are 0.8 km apart. A train accelerates uniformly to a maximum speed when leaving Oxford Circus and maintains this speed for 90 s before decelerating uniformly to stop at Piccadilly Circus. The whole journey takes 2 minutes. Find the maximum speed.

<u>EXAMPLE 3</u>

The diagram shows the velocity–time graph for the motion of a machine's cutting tool. The graph consists of five straight line segments. The tool moves forward for 8 s while cutting and then takes 3 s to return to its starting position.



Find

(i) the acceleration of the tool during the first 2 s of the motion,

(ii) the distance the tool moves forward while cutting,

(iii) the greatest speed of the tool during the return to its starting position.

[Cambridge AS & A Level Mathematics 9709, Paper 41 Q2 June 2010]

EXAMPLE 4

A train travels from A to B, a distance of 20 000 m, taking 1000 s. The journey has three stages. In the first stage the train starts from rest at A and accelerates uniformly until its speed is $V\,\text{m s}^{-1}$. In the second stage the train travels at constant speed $V\,\text{m s}^{-1}$ for 600 s. During the third stage of the journey the train decelerates uniformly, coming to rest at B.

(i) Sketch the velocity–time graph for the train's journey.

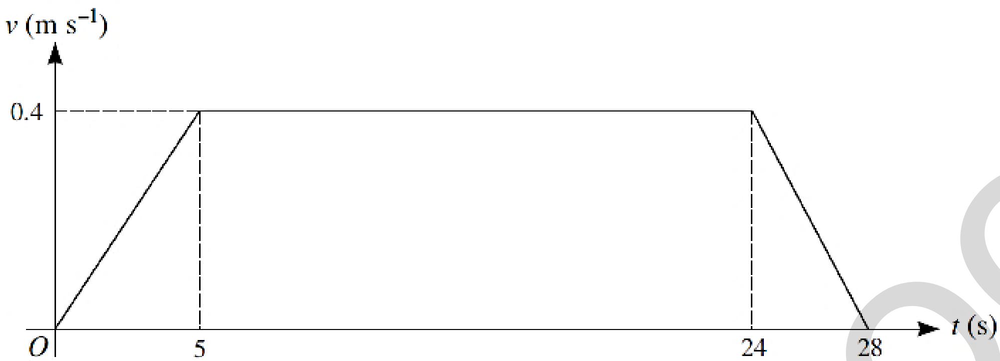
(ii) Find the value of V.

(iii) Given that the acceleration of the train during the first stage of the journey is $0.15\,\text{m s}^{-2}$, find the distance travelled by the train during the third stage of the journey.

[Cambridge AS & A Level Mathematics 9709, Paper 4 Q6 November 2008]

EXAMPLE 5

N13/42/Q7



An elevator is pulled vertically upwards by a cable. The velocity-time graph for the motion is shown above. Find

(i) the distance travelled by the elevator, [2]

(ii) the acceleration during the first stage and the deceleration during the third stage. [2]

EXAMPLE 6

N13/43/Q5

A car travels in a straight line from A to B, a distance of 12 km, taking 552 seconds. The car starts from rest at A and accelerates for T_1 s at 0.3 m s^{-2}, reaching a speed of V m s^{-1}. The car then continues to move at V m s^{-1} for T_2 s. It then decelerates for T_3 s at 1 m s^{-2}, coming to rest at B.

(i) Sketch the velocity-time graph for the motion and express T_1 and T_3 in terms of V. [3]

(ii) Express the total distance travelled in terms of V and show that $13V^2 - 3312V + 72\,000 = 0$. Hence find the value of V. [5]

CONSTANT ACCELERATION AND VERTICAL MOTION UNDER GRAVITY

EXAMPLE 7

A particle starts from a point A with velocity 3 m/s and moves with a constant acceleration of $\frac{1}{2}$ m/s^2 along a straight line AB. It reaches B with a velocity of 5 m/s.

Find (a) the displacement of B from A (b) the time taken from A to B.

EXAMPLE 8

A cyclist starts riding up a straight steep hill with a velocity of 8 m/s. At the top of the hill, which is 96 m long, the velocity is 4 m/s. Assuming a constant acceleration, find its value.

EXAMPLE 9

The driver of a train begins the approach to a station by applying the brakes to produce a steady deceleration of 0.2 m/s^2 and brings the train to rest at the platform in 1 minute 30 seconds.

(a) Find the speed of the train in kilometres/hour at the moment when the brakes were applied,

(b) the distance then travelled before stopping.

EXAMPLE 10

At the same instant two children, who are standing 24 m apart, begin to cycle directly towards each other. James starts from rest at a point A, riding with a constant acceleration of 2 m/s^2 and William rides with a constant speed of 2 m/s. Find how long it is before they meet.

EXAMPLE 11

A coin is dropped from rest at the top of a building of height 12 m and travels in a straight line with constant acceleration 10 m s^{-2}.

Find the time it takes to reach the ground and the speed of impact.

EXAMPLE 12

A particle is projected vertically upwards with a velocity of 30 m s^{-1} from a point O. Find
(a) The maximum height reached,
(b) The time taken for it to return to O,
(c) The time taken for it to be 35 m below O.

EXAMPLE 13

A juggler throws a ball up in the air with initial speed 5 m s^{-1} from a height of 1.2 m. It has a constant acceleration of 10 m s^{-1} vertically downwards due to gravity.

(i) Find the maximum height of the ball above the ground and the time it takes to reach it.

At the instant that the ball reaches its maximum height, the juggler throws up another ball with the same speed and from the same height.

(ii) Where and when will the balls pass each other?

EXAMPLE 14

A particle is projected vertically upwards from a point O with initial speed 12.5 m s^{-1}. At the same instant another particle is released from rest at a point 10 m vertically above O. Find the height above O at which the particles meet.

[Cambridge AS and A Level Mathematics 9709, Paper 4 Q2 November 2007]

EXAMPLE 15

Two points A and B are 8 m apart and lie in the same horizontal plane. A particle passes point A with a speed of 2 m s^{-1} in the direction of point B. The particle is accelerating at a constant rate of 4 m s^{-2} in the direction of its motion. At the same time a second particle is passing point B with a speed of 3 m s^{-1} in the direction of point A. The second particle is accelerating at a constant rate of 2 m s^{-2} in the direction of its motion.

Determine the time in seconds that has passed when the particles meet, and their position when this happens.

KINEMATICS AND CALCULUS

EXAMPLE 16

A particle starts from a point O and moves in a straight line so that its distance s cm from O after time t seconds is given by

$$s = 2t^2 - \frac{t^3}{6}$$

Find

a) its initial velocity and acceleration,
b) the time after the start when it comes to a momentary halt,
c) its distance from O at this time.
d) what maximum velocity does it reach before that time?
e) after what time does the particle pass through O again?

EXAMPLE 17

The distance s meters of a particle moving in a straight line measured from a fixed point O on the line is given by $s = t^2 - 3t + 2$ where t is the time in seconds from the start. Find

a) Its initial distance from O,
b) Its initial velocity and in which direction,
c) Its initial acceleration,
d) The times when it passes through O and with what velocity,
e) When and where it is at instantaneous rest.

EXAMPLE 18

The velocity v cms^{-1} of a particle moving in a straight line is given by $v = 6t - kt^2$, where k is a constant and t is the time in seconds from the start. If its acceleration is 0 when t = 1, find

a) The value of k,
b) The time when the particle comes to instantaneous rest,
c) The maximum velocity of the part;

EXAMPLE 19

The velocity (in $m\,s^{-1}$) of a model train which is moving along straight rails is $v = 0.3t^2 - 0.5$ .Find its displacement from its initial position

a. After time t
b. After 3 seconds.

EXAMPLE 20

A car moves between two sets of traffic lights, stopping at both. Its speed v $m\,s^{-1}$at time t seconds is modelled by

$$v = \frac{1}{20}t(40 - t) \qquad 0 \le t \le 40$$

Find the times at which the car is stationary and the distance between the two of traffic lights.

EXAMPLE 21

The acceleration of a particle (in m s^{-2}) at time t seconds is given by $a = 6 - t$. The particle is initially at the origin with velocity $-2\ ms^{-1}$. Find an expression for:

a) The velocity of the particle after t s
b) The position of the particle after t s.
c) Hence find the velocity and position 6 s later.

EXAMPLE 22

A particle P moves along the x axis in the positive direction. The velocity of P at time t s is $0.03t^2$ ms^{-1}. When t= 5 the displacement of P from the origin O is 2.5m.

a) Find an expression, in terms of t, for the displacement of P from O.
b) Find the velocity of p when its displacement from O is 11.25m.

EXAMPLE 23

J13/42/Q6

A particle P moves in a straight line. It starts from rest at a point O and moves towards a point A on the line. During the first 8 seconds P's speed increases to $8\ \text{m s}^{-1}$ with constant acceleration. During the next 12 seconds P's speed decreases to $2\ \text{m s}^{-1}$ with constant deceleration. P then moves with constant acceleration for 6 seconds, reaching A with speed $6.5\ \text{m s}^{-1}$.

(i) Sketch the velocity-time graph for P's motion. [2]

The displacement of P from O, at time t seconds after P leaves O, is s metres.

(ii) Shade the region of the velocity-time graph representing s for a value of t where $20 \leqslant t \leqslant 26$. [1]

(iii) Show that, for $20 \leqslant t \leqslant 26$,

$$s = 0.375t^2 - 13t + 202.$$ [6]

EXAMPLE 24

J12/41/Q4

A particle P starts at the point O and travels in a straight line. At time t seconds after leaving O the velocity of P is $v\ \text{m s}^{-1}$, where $v = 0.75t^2 - 0.0625t^3$. Find

(i) the positive value of t for which the acceleration is zero, [3]

(ii) the distance travelled by P before it changes its direction of motion. [5]

Example 25

J12/42/Q3

A particle P moves in a straight line, starting from the point O with velocity 2 m s^{-1}. The acceleration of P at time t s after leaving O is $2t^{\frac{2}{3}}\text{ m s}^{-2}$.

(i) Show that $t^{\frac{5}{3}} = \frac{5}{6}$ when the velocity of P is 3 m s^{-1}. [4]

(ii) Find the distance of P from O when the velocity of P is 3 m s^{-1}. [3]

Example 26

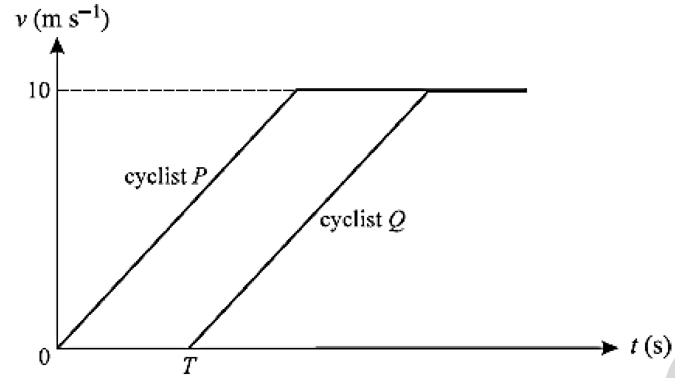
A particle moves on a straight line through O so that its displacement s from O at time t seconds is given by the equation $s = 4 + 12t^2 - t^3$. Find the total distance covered in the first 12 seconds. What will be the acceleration of the particle when it is momentarily at rest?

Example 27

If the acceleration of a particle that moves along a straight line through a point O at time t is given by the equation $a = 2t^3 - 3t^2$ and the velocity of the particle when $t = 2$ is 10 m s^{-1}, then find an expression for the velocity.

HOMEWORK: KINEMATICS VARIANT 42

1



The diagram shows the velocity-time graphs for the motion of two cyclists P and Q, who travel in the same direction along a straight path. Both cyclists start from rest at the same point O and both accelerate at $2\,\text{m}\,\text{s}^{-2}$ up to a speed of $10\,\text{m}\,\text{s}^{-1}$. Both then continue at a constant speed of $10\,\text{m}\,\text{s}^{-1}$. Q starts his journey T seconds after P.

(i) Show in a sketch of the diagram the region whose area represents the displacement of P, from O, at the instant when Q starts. [1]

Given that P has travelled 16 m at the instant when Q starts, find

(ii) the value of T, [3]

(iii) the distance between P and Q when Q's speed reaches $10\,\text{m}\,\text{s}^{-1}$. [2]

Answers: (ii) 4; (iii) 40 m. **J03/Q3**

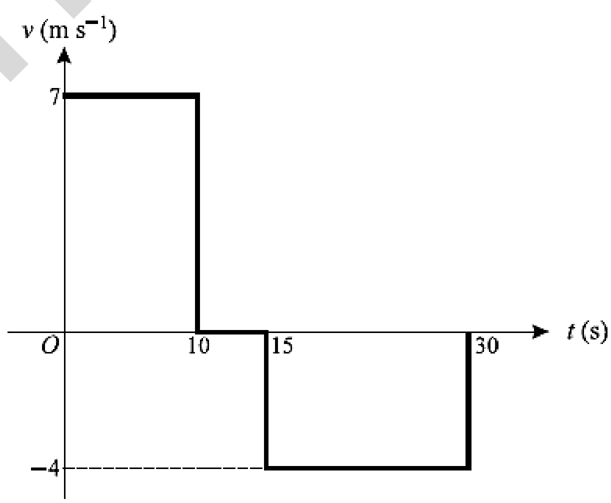
2 A particle moves in a straight line. Its displacement t seconds after leaving the fixed point O is x metres, where $x = \frac{1}{2}t^2 + \frac{1}{30}t^3$. Find

(i) the speed of the particle when $t = 10$, [3]

(ii) the value of t for which the acceleration of the particle is twice its initial acceleration. [3]

Answers: (i) $20\,\text{ms}^{-1}$; (ii) 5. **J03/Q4**

3



A boy runs from a point A to a point C. He pauses at C and then walks back towards A until reaching the point B, where he stops. The diagram shows the graph of v against t, where v m s^{-1} is the boy's velocity at time t seconds after leaving A. The boy runs and walks in the same straight line throughout.

(i) Find the distances AC and AB. [3]

(ii) Sketch the graph of x against t, where x metres is the boy's displacement from A. Show clearly the values of t and x when the boy arrives at C, when he leaves C, and when he arrives at B. [3]

Answers: (i) 70 m, 10 m. **J04/Q3**

4 A particle P moves in a straight line that passes through the origin O. The velocity of P at time t seconds is v m s^{-1}, where $v = 20t - t^3$. At time $t = 0$ the particle is at rest at a point whose displacement from O is -36 m.

(i) Find an expression for the displacement of P from O in terms of t. [3]

(ii) Find the displacement of P from O when $t = 4$. [1]

(iii) Find the values of t for which the particle is at O. [3]

Answers: (i) $10t^2 - 0.25t^4 - 36$; (ii) 60 m; (iii) 2, 6. **J04/Q5**

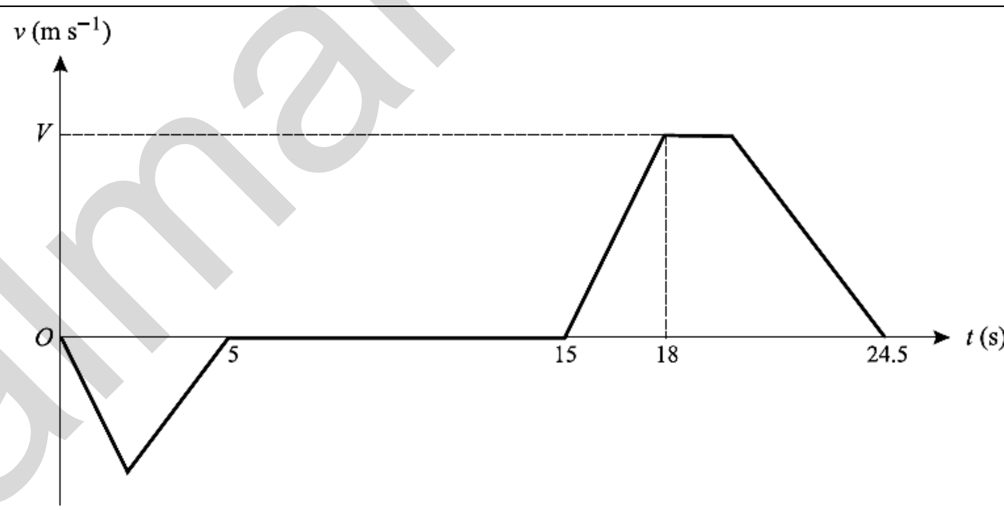
5 A particle P moves along the x-axis in the positive direction. The velocity of P at time t s is $0.03t^2$ m s^{-1}. When $t = 5$ the displacement of P from the origin O is 2.5 m.

(i) Find an expression, in terms of t, for the displacement of P from O. [4]

(ii) Find the velocity of P when its displacement from O is 11.25 m. [3]

Answers: (i) $0.01t^3 + 1.25$; (ii) 3 ms^{-1}. **J05/Q5**

6



The diagram shows the velocity-time graph for a lift moving between floors in a building. The graph consists of straight line segments. In the first stage the lift travels downwards from the ground floor for 5 s, coming to rest at the basement after travelling 10 m.

(i) Find the greatest speed reached during this stage. [2]

The second stage consists of a 10 s wait at the basement. In the third stage, the lift travels upwards until it comes to rest at a floor 34.5 m above the basement, arriving 24.5 s after the start of the first stage. The lift accelerates at $2\,\text{m s}^{-2}$ for the first 3 s of the third stage, reaching a speed of $V\,\text{m s}^{-1}$. Find

(ii) the value of V, [2]

(iii) the time during the third stage for which the lift is moving at constant speed, [3]

(iv) the deceleration of the lift in the final part of the third stage. [2]

Answers: (i) $4\,\text{ms}^{-1}$; (ii) 6; (iii) 2 s; (iv) $\frac{4}{3}\,\text{ms}^{-2}$. **J05/Q6**

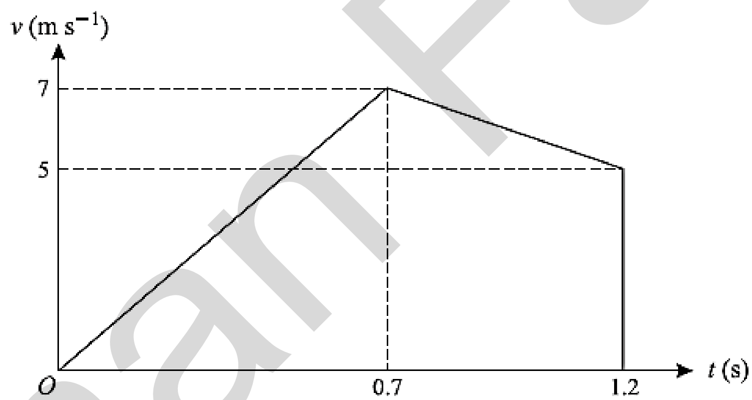
7 A motorcyclist starts from rest at A and travels in a straight line until he comes to rest again at B. The velocity of the motorcyclist t seconds after leaving A is $v\,\text{m s}^{-1}$, where $v = t - 0.01t^2$. Find

(i) the time taken for the motorcyclist to travel from A to B, [2]

(ii) the distance AB. [3]

Answers: (i) 100 s; (ii) 1670 m. **J06/Q2**

8



The diagram shows the velocity-time graph for the motion of a small stone which falls vertically from rest at a point A above the surface of liquid in a container. The downward velocity of the stone t s after leaving A is $v\,\text{m s}^{-1}$. The stone hits the surface of the liquid with velocity $7\,\text{m s}^{-1}$ when $t = 0.7$. It reaches the bottom of the container with velocity $5\,\text{m s}^{-1}$ when $t = 1.2$.

(i) Find

(a) the height of A above the surface of the liquid,

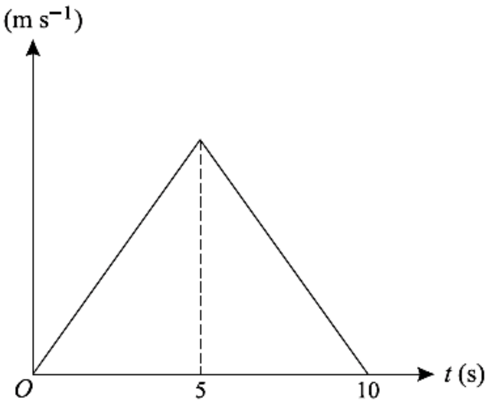
(b) the depth of liquid in the container.

[3]

(ii) Find the deceleration of the stone while it is moving in the liquid. [2]

Answers: (i)(a) 2.4 m, (b) 3 m; (ii) $4\,\text{ms}^{-2}$; (iii) 0.05 kg. **J06/Q4**

9



A particle P starts from rest at the point A and travels in a straight line, coming to rest again after 10 s. The velocity-time graph for P consists of two straight line segments (see diagram). A particle Q starts from rest at A at the same instant as P and travels along the same straight line as P. The velocity of Q is given by $v = 3t - 0.3t^2$ for $0 \leqslant t \leqslant 10$. The displacements from A of P and Q are the same when $t = 10$.

(i) Show that the greatest velocity of P during its motion is $10\,\text{m s}^{-1}$. [6]

(ii) Find the value of t, in the interval $0 < t < 5$, for which the acceleration of Q is the same as the acceleration of P. [3]

J07/Q6

Answer: (ii) $1\frac{2}{3}$.

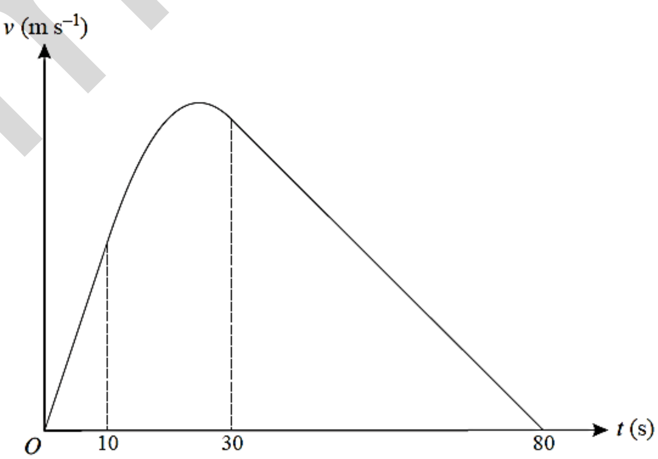
10 A particle P of mass 0.6 kg is projected vertically upwards with speed $5.2\,\text{m s}^{-1}$ from a point O which is 6.2 m above the ground. Air resistance acts on P so that its deceleration is $10.4\,\text{m s}^{-2}$ when P is moving upwards, and its acceleration is $9.6\,\text{m s}^{-2}$ when P is moving downwards. Find

(i) the greatest height above the ground reached by P, [3]

(ii) the speed with which P reaches the ground, [2]

Answers: (i) 7.5 m; (ii) $12\,\text{ms}^{-1}$; (iii) 2.11 J. **J08/Q6**

11



An object P travels from A to B in a time of 80 s. The diagram shows the graph of v against t, where $v\,\text{m s}^{-1}$ is the velocity of P at time t s after leaving A. The graph consists of straight line segments for the intervals $0 \leqslant t \leqslant 10$ and $30 \leqslant t \leqslant 80$, and a curved section whose equation is $v = -0.01t^2 + 0.5t - 1$ for $10 \leqslant t \leqslant 30$. Find

(i) the maximum velocity of P, [4]

(ii) the distance AB. [9]

Answers: (i) 5.25 ms^{-1}; (ii) 233 m. J08/Q7

12 A particle P travels in a straight line from A to D, passing through the points B and C. For the section AB the velocity of the particle is $(0.5t - 0.01t^2)\,\text{m s}^{-1}$, where t s is the time after leaving A.

(i) Given that the acceleration of P at B is $0.1\,\text{m s}^{-2}$, find the time taken for P to travel from A to B. [3]

The acceleration of P from B to C is constant and equal to $0.1\,\text{m s}^{-2}$.

(ii) Given that P reaches C with speed $14\,\text{m s}^{-1}$, find the time taken for P to travel from B to C. [3]

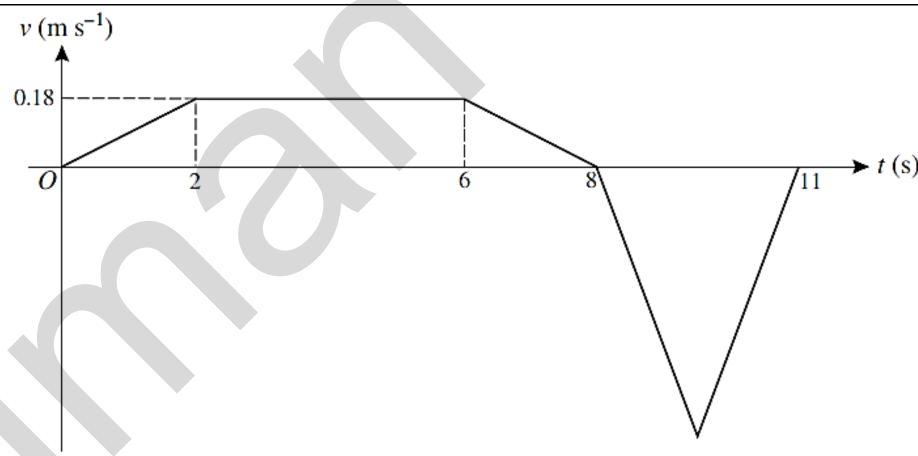
P travels with constant deceleration $0.3\,\text{m s}^{-2}$ from C to D. Given that the distance CD is 300 m, find

(iii) the speed with which P reaches D, [2]

(iv) the distance AD. [6]

Answers: (i) 20 s; (ii) 80 s; (iii) 4 ms^{-1}; (iv) 1170 m. J09/Q7

13



The diagram shows the velocity-time graph for the motion of a machine's cutting tool. The graph consists of five straight line segments. The tool moves forward for 8 s while cutting and then takes 3 s to return to its starting position. Find

(i) the acceleration of the tool during the first 2 s of the motion, [1]

(ii) the distance the tool moves forward while cutting, [2]

(iii) the greatest speed of the tool during the return to its starting position. [2]

Answers: (i) 20 s; (ii) 80 s; (iii) 4 ms^{-1}; (iv) 1170 m. J10/42/Q2

14 A vehicle is moving in a straight line. The velocity $v\,\text{m s}^{-1}$ at time t s after the vehicle starts is given by

$$v = A(t - 0.05t^2) \quad \text{for } 0 \leqslant t \leqslant 15,$$
$$v = \frac{B}{t^2} \quad \text{for } t \geqslant 15,$$

where A and B are constants. The distance travelled by the vehicle between $t = 0$ and $t = 15$ is 225 m.

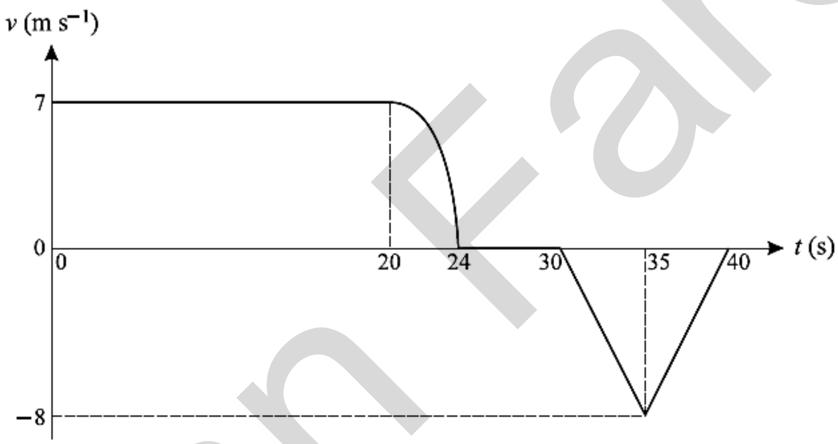
(i) Find the value of A and show that $B = 3375$. [5]

(ii) Find an expression in terms of t for the total distance travelled by the vehicle when $t \geqslant 15$. [3]

(iii) Find the speed of the vehicle when it has travelled a total distance of 315 m. [3]

Answers: (i) 4; (ii) $(450 - \frac{3375}{t})$ m; (iii) 5.4 ms^{-1}. **J10/42/Q7**

15



A man runs in a straight line. He passes through a fixed point A with constant velocity $7\,\text{m s}^{-1}$ at time $t = 0$. At time t s his velocity is $v\,\text{m s}^{-1}$. The diagram shows the graph of v against t for the period $0 \leqslant t \leqslant 40$.

(i) Show that the man runs more than 154 m in the first 24 s. [2]

(ii) Given that the man runs 20 m in the interval $20 \leqslant t \leqslant 24$, find how far he is from A when $t = 40$. [2]

Answer: **(ii)** 120 m. **N02/Q2**

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16 Two particles A and B are projected vertically upwards from horizontal ground at the same instant. The speeds of projection of A and B are $5\,\text{m s}^{-1}$ and $8\,\text{m s}^{-1}$ respectively. Find

(i) the difference in the heights of A and B when A is at its maximum height, [4]

(ii) the height of A above the ground when B is 0.9 m above A. [4]

Answers: **(ii)** 162; **(iii)** 688 747 536 or 689 000 000. **N02/Q4**

17 A particle P starts to move from a point O and travels in a straight line. At time t s after P starts to move its velocity is $v \text{ m s}^{-1}$, where $v = 0.12t - 0.0006t^2$.

(i) Verify that P comes to instantaneous rest when $t = 200$, and find the acceleration with which it starts to return towards O. [3]

(ii) Find the maximum speed of P for $0 \leqslant t \leqslant 200$. [3]

(iii) Find the displacement of P from O when $t = 200$. [3]

(iv) Find the value of t when P reaches O again. [2]

Answers: **(i)** 0.12 ms^{-2}; **(ii)** 6 ms^{-1}; **(iii)** 800 m; **(iv)** 300. **N02/Q7**

18 A stone is released from rest and falls freely under gravity. Find

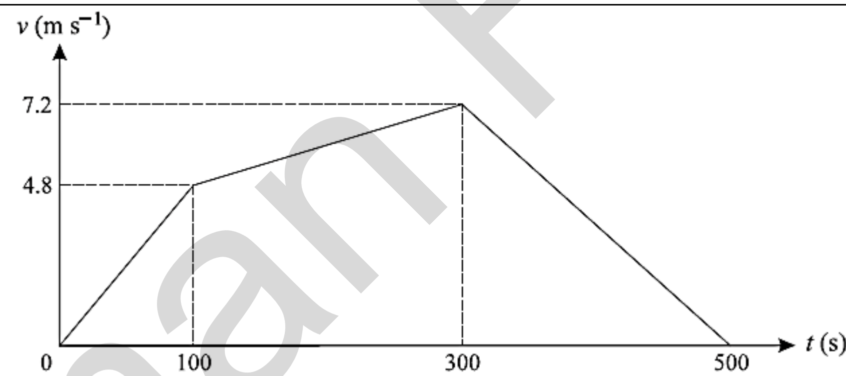
(i) the speed of the stone after 2 s, [1]

(ii) the time taken for the stone to fall a distance of 45 m from its initial position, [2]

(iii) the distance fallen by the stone from the instant when its speed is 30 m s^{-1} to the instant when its speed is 40 m s^{-1}. [2]

Answers: **(i)** 20 ms^{-1}; **(ii)** 3 s; **(iii)** 35 m. **N03/Q2**

19



A tractor A starts from rest and travels along a straight road for 500 seconds. The velocity-time graph for the journey is shown above. This graph consists of three straight line segments. Find

(i) the distance travelled by A, [3]

(ii) the initial acceleration of A. [2]

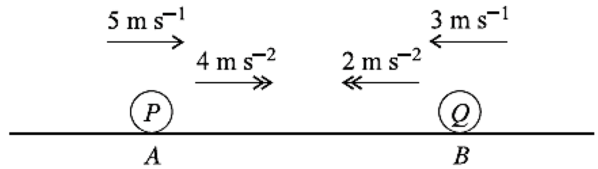
Another tractor B starts from rest at the same instant as A, and travels along the same road for 500 seconds. Its velocity t seconds after starting is $(0.06t - 0.00012t^2) \text{ m s}^{-1}$. Find

(iii) how much greater B's initial acceleration is than A's, [2]

(iv) how much further B has travelled than A, at the instant when B's velocity reaches its maximum. [6]

Answers: **(i)** 2160 m; **(ii)** 0.048 ms^{-2}; **(iii)** 0.012 ms^{-2}; **(iv)** 155 m. **N03/Q7**

20



Particles P and Q start from points A and B respectively, at the same instant, and move towards each other in a horizontal straight line. The initial speeds of P and Q are $5\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively. The accelerations of P and Q are constant and equal to $4\,\text{m s}^{-2}$ and $2\,\text{m s}^{-2}$ respectively (see diagram).

(i) Find the speed of P at the instant when the speed of P is 1.8 times the speed of Q. [4]

(ii) Given that $AB = 51$ m, find the time taken from the start until P and Q meet. [4]

Answers: (i) 9 ms^{-1}; (ii) 3 s. **N04/Q5**

21 A particle starts from rest at the point A and travels in a straight line until it reaches the point B. The velocity of the particle t seconds after leaving A is $v\,\text{m s}^{-1}$, where $v = 0.009t^2 - 0.0001t^3$. Given that the velocity of the particle when it reaches B is zero, find

(i) the time taken for the particle to travel from A to B, [2]

(ii) the distance AB, [4]

(iii) the maximum velocity of the particle. [4]

Answers: (i) 90 s; (ii) 547 m; (iii) 10.8 ms^{-1}. **N04/Q7**

22 A car travels in a straight line with constant acceleration $a\,\text{m s}^{-2}$. It passes the points A, B and C, in this order, with speeds $5\,\text{m s}^{-1}$, $7\,\text{m s}^{-1}$ and $8\,\text{m s}^{-1}$ respectively. The distances AB and BC are d_1 m and d_2 m respectively.

(i) Write down an equation connecting

(a) d_1 and a,

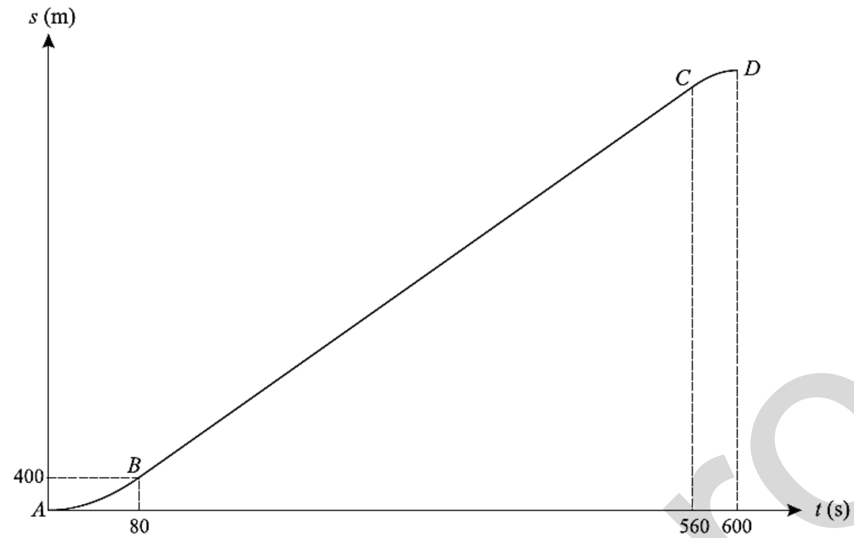
(b) d_2 and a.

[2]

(ii) Hence find d_1 in terms of d_2. [2]

N05/Q1

23



The diagram shows the displacement-time graph for a car's journey. The graph consists of two curved parts AB and CD, and a straight line BC. The line BC is a tangent to the curve AB at B and a tangent to the curve CD at C. The gradient of the curves at $t = 0$ and $t = 600$ is zero, and the acceleration of the car is constant for $0 < t < 80$ and for $560 < t < 600$. The displacement of the car is 400 m when $t = 80$.

(i) Sketch the velocity-time graph for the journey. [3]

(ii) Find the velocity at $t = 80$. [2]

(iii) Find the total distance for the journey. [2]

(iv) Find the acceleration of the car for $0 < t < 80$. [2]

N05/Q5

24 A particle P starts from rest at O and travels in a straight line. Its velocity $v\,\text{m s}^{-1}$ at time t s is given by $v = 8t - 2t^2$ for $0 \leqslant t \leqslant 3$, and $v = \frac{54}{t^2}$ for $t > 3$. Find

(i) the distance travelled by P in the first 3 seconds, [4]

(ii) an expression in terms of t for the displacement of P from O, valid for $t > 3$, [3]

(iii) the value of v when the displacement of P from O is 27 m. [3]

N05/Q6

25 The velocity of a particle t s after it starts from rest is $v\,\text{m s}^{-1}$, where $v = 1.25t - 0.05t^2$. Find

(i) the initial acceleration of the particle, [2]

(ii) the displacement of the particle from its starting point at the instant when its acceleration is $0.05\,\text{m s}^{-2}$. [5]

Answers: (i) $1.25\ \text{ms}^{-2}$; (ii) 61.2 m.

N06/Q4

26 **(i)** A man walks in a straight line from A to B with constant acceleration $0.004\,\text{m s}^{-2}$. His speed at A is $1.8\,\text{m s}^{-1}$ and his speed at B is $2.2\,\text{m s}^{-1}$. Find the time taken for the man to walk from A to B, and find the distance AB. [3]

(ii) A woman cyclist leaves A at the same instant as the man. She starts from rest and travels in a straight line to B, reaching B at the same instant as the man. At time t s after leaving A the cyclist's speed is $k(200t - t^2)\,\text{m s}^{-1}$, where k is a constant. Find

(a) the value of k, [4]

(b) the cyclist's speed at B. [1]

(iii) Sketch, using the same axes, the velocity-time graphs for the man's motion and the woman's motion from A to B. [3]

Answers: (i) 100 s, 200 m; (ii)(a) 0.0003, (b) 3 ms^{-1}. **N07/Q6**

---

27 A train travels from A to B, a distance of 20 000 m, taking 1000 s. The journey has three stages. In the first stage the train starts from rest at A and accelerates uniformly until its speed is $V\,\text{m s}^{-1}$. In the second stage the train travels at constant speed $V\,\text{m s}^{-1}$ for 600 s. During the third stage of the journey the train decelerates uniformly, coming to rest at B.

(i) Sketch the velocity-time graph for the train's journey. [2]

(ii) Find the value of V. [3]

(iii) Given that the acceleration of the train during the first stage of the journey is $0.15\,\text{m s}^{-2}$, find the distance travelled by the train during the third stage of the journey. [4]

Answers: (ii) 25; (iii) 2920 m. **N08/Q6**

---

28 A particle P is held at rest at a fixed point O and then released. P falls freely under gravity until it reaches the point A which is 1.25 m below O.

(i) Find the speed of P at A and the time taken for P to reach A. [3]

The particle continues to fall, but now its downward acceleration t seconds after passing through A is $(10 - 0.3t)\,\text{m s}^{-2}$.

(ii) Find the total distance P has fallen, 3 s after being released from O. [7]

Answers: (i) 5 ms^{-1}, 0.5 s; (ii) 44.2 m. **N08/Q7**

---

29 A motorcyclist starts from rest at A and travels in a straight line. For the first part of the motion, the motorcyclist's displacement x metres from A after t seconds is given by $x = 0.6t^2 - 0.004t^3$.

(i) Show that the motorcyclist's acceleration is zero when $t = 50$ and find the speed $V\,\text{m s}^{-1}$ at this time. [5]

For $t \geqslant 50$, the motorcyclist travels at constant speed $V\,\text{m s}^{-1}$.

(ii) Find the value of t for which the motorcyclist's average speed is $27.5\,\text{m s}^{-1}$. [5]

Answers: **(i)** 30 ms^{-1}; **(ii)** 200.

N09/42/Q7

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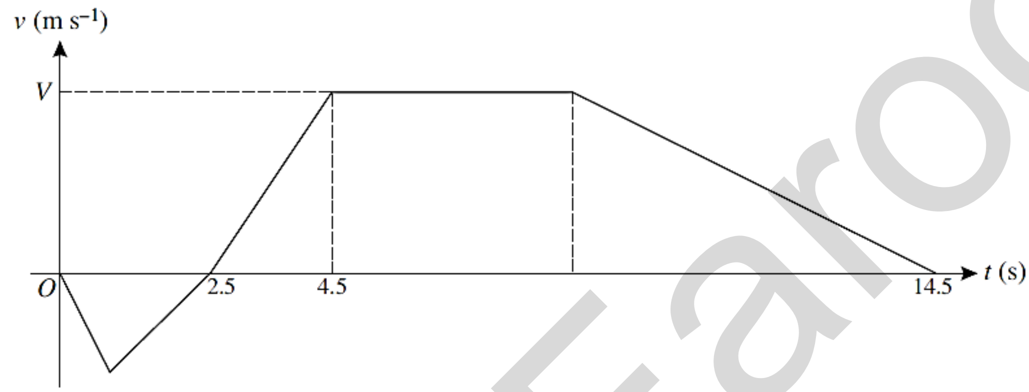
30 Particles P and Q are projected vertically upwards, from different points on horizontal ground, with velocities of 20 m s^{-1} and 25 m s^{-1} respectively. Q is projected 0.4 s later than P. Find

(i) the time for which P's height above the ground is greater than 15 m, [3]

(ii) the velocities of P and Q at the instant when the particles are at the same height. [5]

N10/42/Q5

31



The diagram shows the velocity-time graph for a particle P which travels on a straight line AB, where v m s^{-1} is the velocity of P at time t s. The graph consists of five straight line segments. The particle starts from rest when $t = 0$ at a point X on the line between A and B and moves towards A. The particle comes to rest at A when $t = 2.5$.

(i) Given that the distance XA is 4 m, find the greatest speed reached by P during this stage of the motion. [2]

In the second stage, P starts from rest at A when $t = 2.5$ and moves towards B. The distance AB is 48 m. The particle takes 12 s to travel from A to B and comes to rest at B. For the first 2 s of this stage P accelerates at 3 m s^{-2}, reaching a velocity of V m s^{-1}. Find

(ii) the value of V, [2]

(iii) the value of t at which P starts to decelerate during this stage, [3]

(iv) the deceleration of P immediately before it reaches B. [2]

N10/42/Q6

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32 A particle P travels in a straight line. It passes through the point O of the line with velocity 5 m s^{-1} at time $t = 0$, where t is in seconds. P's velocity after leaving O is given by

$$(0.002t^3 - 0.12t^2 + 1.8t + 5) \text{ m s}^{-1}.$$

The velocity of P is increasing when $0 < t < T_1$ and when $t > T_2$, and the velocity of P is decreasing when $T_1 < t < T_2$.

(i) Find the values of T_1 and T_2 and the distance OP when $t = T_2$. [7]

(ii) Find the velocity of P when $t = T_2$ and sketch the velocity-time graph for the motion of P. [3]

N10/42/Q7

33 A particle P_1 is projected vertically upwards, from horizontal ground, with a speed of $30\,\mathrm{m\,s^{-1}}$. At the same instant another particle P_2 is projected vertically upwards from the top of a tower of height 25 m, with a speed of $10\,\mathrm{m\,s^{-1}}$. Find

(i) the time for which P_1 is higher than the top of the tower, [3]

(ii) the velocities of the particles at the instant when the particles are at the same height, [5]

(iii) the time for which P_1 is higher than P_2 and is moving upwards. [3]

Answers: (i) 4 s; (ii) 17.5 $\mathrm{ms^{-1}}$ and –2.5 $\mathrm{ms^{-1}}$; (iii) 1.75 s. **J04/Q7**

34 A particle is projected vertically upwards from a point O with initial speed $12.5\,\mathrm{m\,s^{-1}}$. At the same instant another particle is released from rest at a point 10 m vertically above O. Find the height above O at which the particles meet. [5]

Answer. 6.8 m. **N07/Q2**

35 Two particles P and Q are projected vertically upwards from horizontal ground at the same instant. The speeds of projection of P and Q are $12\,\mathrm{m\,s^{-1}}$ and $7\,\mathrm{m\,s^{-1}}$ respectively and the heights of P and Q above the ground, t seconds after projection, are h_P m and h_Q m respectively. Each particle comes to rest on returning to the ground.

(i) Find the set of values of t for which the particles are travelling in opposite directions. [3]

(ii) At a certain instant, P and Q are above the ground and $3h_P = 8h_Q$. Find the velocities of P and Q at this instant. [5]

Answers. (i) $0.7 < t < 1.2$; (ii) 4 $\mathrm{ms^{-1}}$, –1 $\mathrm{ms^{-1}}$. **J11/42/Q5**

36 A walker travels along a straight road passing through the points A and B on the road with speeds $0.9\,\mathrm{m\,s^{-1}}$ and $1.3\,\mathrm{m\,s^{-1}}$ respectively. The walker's acceleration between A and B is constant and equal to $0.004\,\mathrm{m\,s^{-2}}$.

(i) Find the time taken by the walker to travel from A to B, and find the distance AB. [3]

A cyclist leaves A at the same instant as the walker. She starts from rest and travels along the straight road, passing through B at the same instant as the walker. At time t s after leaving A the cyclist's speed is $kt^3\,\mathrm{m\,s^{-1}}$, where k is a constant.

(ii) Show that when $t = 64.05$ the speed of the walker and the speed of the cyclist are the same, correct to 3 significant figures. [5]

(ii) Find the cyclist's acceleration at the instant she passes through B. [2]

Answers. (i) 100 s, 110 m; (iii) 0.132 $\mathrm{ms^{-2}}$. **J11/42/Q7**

37 A particle P moves in a straight line, starting from the point O with velocity $2\,\mathrm{m\,s^{-1}}$. The acceleration of P at time t s after leaving O is $2t^{\frac{2}{3}}\,\mathrm{m\,s^{-2}}$.

(i) Show that $t^{\frac{5}{3}} = \frac{5}{6}$ when the velocity of P is $3\,\mathrm{m\,s^{-1}}$. [4]

(ii) Find the distance of P from O when the velocity of P is $3\,\mathrm{m\,s^{-1}}$. [3]

Answers: **(i)** $t^{5/3} = 5/6$ **(ii)** $OP = 2.13$ m **J12/42/Q3**

38 A particle P moves in a straight line. It starts from rest at a point O and moves towards a point A on the line. During the first 8 seconds P's speed increases to 8 m s^{-1} with constant acceleration. During the next 12 seconds P's speed decreases to 2 m s^{-1} with constant deceleration. P then moves with constant acceleration for 6 seconds, reaching A with speed 6.5 m s^{-1}.

(i) Sketch the velocity-time graph for P's motion. [2]

The displacement of P from O, at time t seconds after P leaves O, is s metres.

(ii) Shade the region of the velocity-time graph representing s for a value of t where $20 \leqslant t \leqslant 26$. [1]

(iii) Show that, for $20 \leqslant t \leqslant 26$,

$$s = 0.375t^2 - 13t + 202.$$ [6]

J13/42/Q6

39 A and B are two points which are 10 m apart on the same horizontal plane. A particle P starts to move from rest at A, directly towards B, with constant acceleration 0.5 m s^{-2}. Another particle Q is moving directly towards A with constant speed 0.75 m s^{-1}, and passes through B at the instant that P starts to move. At time T s after this instant, particles P and Q collide. Find

(i) the value of T, [4]

(ii) the speed of P immediately before the collision. [1]

Answer: $T = 5$ *Answer:* Speed of $P = 2.5\text{ ms}^{-1}$ **J14/42/Q2**

40 A particle P moves in a straight line. At time t seconds after starting from rest at the point O on the line, the acceleration of P is $a\text{ m s}^{-2}$, where $a = 0.075t^2 - 1.5t + 5$.

(i) Find an expression for the displacement of P from O in terms of t. [4]

(ii) Hence find the time taken for P to return to the point O. [3]

Answer: **(i)** The displacement of P from O is = $0.00625t^4 - 0.25t^3 + 2.5t^2$ **J15/42/Q4**

or equivalent forms such as displacement of P from O = $\frac{1}{160}t^4 - \frac{1}{4}t^3 + \frac{5}{2}t^2$

(ii) Time taken for P to return to the point O is 20 s

41 A particle P starts from rest at a point O on a horizontal straight line. P moves along the line with constant acceleration and reaches a point A on the line with a speed of 30 m s^{-1}. At the instant that P leaves O, a particle Q is projected vertically upwards from the point A with a speed of 20 m s^{-1}. Subsequently P and Q collide at A. Find

(i) the acceleration of P, [4]

(ii) the distance OA. [2]

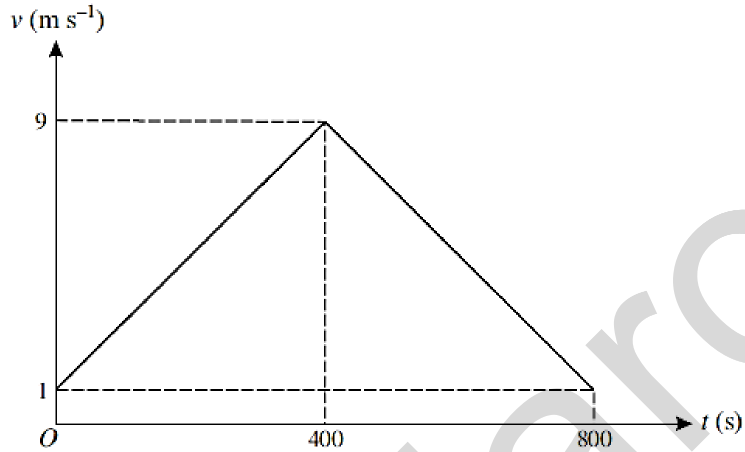
Answer: **(i)** The acceleration of P is 7.5 ms^{-2} **(ii)** The distance OA is 60 m **J15/42/Q5**

42 A particle P moves in a straight line. It starts from a point O on the line with velocity 1.8 m s^{-1}. The acceleration of P at time t s after leaving O is $0.8t^{-0.75}\text{ m s}^{-2}$. Find the displacement of P from O when $t = 16$. [6]

Answers: 111 m **N11/42/Q3**

43 A tractor travels in a straight line from a point A to a point B. The velocity of the tractor is $v\,\text{m s}^{-1}$ at time t s after leaving A.

(i)



The diagram shows an approximate velocity-time graph for the motion of the tractor. The graph consists of two straight line segments. Use the graph to find an approximation for

(a) the distance AB, [2]

(b) the acceleration of the tractor for $0 < t < 400$ and for $400 < t < 800$. [2]

(ii) The actual velocity of the tractor is given by $v = 0.04t - 0.00005t^2$ for $0 \leqslant t \leqslant 800$.

(a) Find the values of t for which the actual acceleration of the tractor is given correctly by the approximate velocity-time graph in part **(i)**. [3]

For the interval $0 \leqslant t \leqslant 400$, the approximate velocity of the tractor in part (i) is denoted by $v_1\,\text{m s}^{-1}$.

(b) Express v_1 in terms of t and hence show that $v_1 - v = 0.00005(t - 200)^2 - 1$. [2]

(c) Deduce that $-1 \leqslant v_1 - v \leqslant 1$. [2]

Answers: (i) (a) 4000 m; (b) 0.02 ms^{-2}, – 0.02 ms^{-2}; (ii) (a) 200, 600 **N11/42/Q7**

44 A car travels along a straight road with constant acceleration $a\,\text{m s}^{-2}$. It passes through points A, B and C; the time taken from A to B and from B to C is 5 s in each case. The speed of the car at A is $u\,\text{m s}^{-1}$ and the distances AB and BC are 55 m and 65 m respectively. Find the values of a and u. [6]

Answer: a = 0.4, u = 10. **N12/42/Q3**

45 A particle P starts to move from a point O and travels in a straight line. The velocity of P is $k(60t^2 - t^3)\,\text{m s}^{-1}$ at time t s after leaving O, where k is a constant. The maximum velocity of P is $6.4\,\text{m s}^{-1}$.

(i) Show that $k = 0.0002$. [3]

P comes to instantaneous rest at a point A on the line. Find

(ii) the distance OA, [5]

(iii) the magnitude of the acceleration of P at A, [2]

(iv) the speed of P when it subsequently passes through O. [2]

Answers: **(ii)** 216 m; **(iii)** 0.72 ms^{-2}; **(iv)** 25.6 ms^{-1}. **N12/42/Q7**

46 A particle P moves in a straight line. P starts from rest at O and travels to A where it comes to rest, taking 50 seconds. The speed of P at time t seconds after leaving O is v m s^{-1}, where v is defined as follows.

$$\text{For } 0 \leqslant t \leqslant 5, \quad v = t - 0.1t^2,$$
$$\text{for } 5 \leqslant t \leqslant 45, \quad v \text{ is constant},$$
$$\text{for } 45 \leqslant t \leqslant 50, \quad v = 9t - 0.1t^2 - 200.$$

(i) Find the distance travelled by P in the first 5 seconds. [3]

(ii) Find the total distance from O to A, and deduce the average speed of P for the whole journey from O to A. [6]

Answer: Distance = 8.33 m **N13/42/Q5**
Answer: Total distance from O to A = 117 m, Average speed = 2.33 ms^{-1}

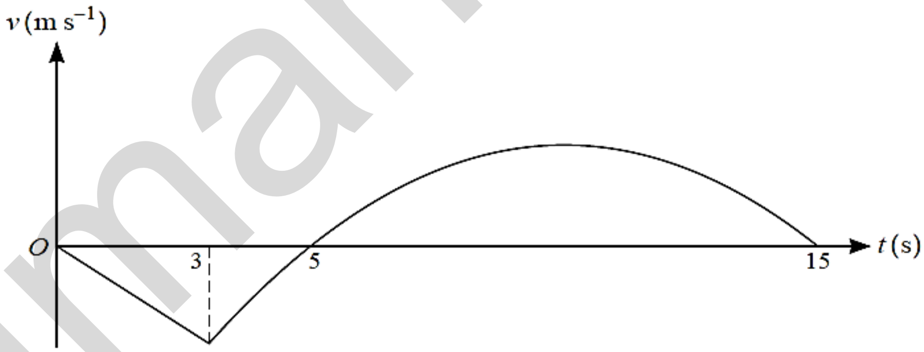
47 A particle P is projected vertically upwards with speed 11 m s^{-1} from a point on horizontal ground. At the same instant a particle Q is released from rest at a point h m above the ground. P and Q hit the ground at the same instant, when Q has speed V m s^{-1}.

(i) Find the time after projection at which P hits the ground. [2]

(ii) Hence find the values of h and V. [2]

Answer: 2.2, 24.2, 22 **N14/42/Q1**

48



The diagram shows the velocity-time graph for the motion of a particle P which moves on a straight line BAC. It starts at A and travels to B taking 5 s. It then reverses direction and travels from B to C taking 10 s. For the first 3 s of P's motion its acceleration is constant. For the remaining 12 s the velocity of P is v m s^{-1} at time t s after leaving A, where

$$v = -0.2t^2 + 4t - 15 \quad \text{for } 3 \leqslant t \leqslant 15.$$

(i) Find the value of v when $t = 3$ and the magnitude of the acceleration of P for the first 3 s of its motion. [3]

(ii) Find the maximum velocity of P while it is moving from B to C. [3]

(iii) Find the average speed of P,

(a) while moving from A to B,

(b) for the whole journey. [6]

Answer: -4.8, 1.6, 5, 2.35, 3.00 **N14/42/Q7**

49 A particle is released from rest at a point H m above horizontal ground and falls vertically. The particle passes through a point 35 m above the ground with a speed of $(V - 10)\text{ m s}^{-1}$ and reaches the ground with a speed of $V\text{ m s}^{-1}$. Find

(i) the value of V, [3]

(ii) the value of H. [2]

Answer: $V = 40$ *Answer*: $H = 80$ **N15/42/Q2**

50 A particle P moves along a straight line for 100 s. It starts at a point O and at time t seconds after leaving O the velocity of P is $v\text{ m s}^{-1}$, where

$$v = 0.000\,04t^3 - 0.006t^2 + 0.288t.$$

(i) Find the values of t at which the acceleration of P is zero. [3]

(ii) Find the displacement of P from O when $t = 100$. [3]

Answer: Values of t at which acceleration is zero are $t = 40$ and $t = 60$ **N15/42/Q3**

Answer: The displacement of P from O when $t = 100$ is 440 m

51 A particle P moves on a straight line, starting from rest at a point O of the line. The time after P starts to move is t s, and the particle moves along the line with constant acceleration $\frac{1}{4}\text{ m s}^{-2}$ until it passes through a point A at time $t = 8$. After passing through A the velocity of P is $\frac{1}{2}t^{\frac{2}{3}}\text{ m s}^{-1}$.

(i) Find the acceleration of P immediately after it passes through A. Hence show that the acceleration of P decreases by $\frac{1}{12}\text{ m s}^{-2}$ as it passes through A. [4]

(ii) Find the distance moved by P from $t = 0$ to $t = 27$. [3]

Answer: Acceleration of $P = \frac{t^{-\frac{1}{3}}}{3}$ for $8 \le t \le 27$ with $a = \frac{1}{6}$ at $t = 8$. Decrease $= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ **J14/42/Q4**

Answer: Distance moved by $P = 71.3$ m

HOMEWORK: KINEMATICS VARIANT 41 & 43

1 A particle P moves on a straight line. It starts at a point O on the line and returns to O 100 s later. The velocity of P is $v\,\text{m s}^{-1}$ at time t s after leaving O, where

$$v = 0.0001t^3 - 0.015t^2 + 0.5t.$$

(i) Show that P is instantaneously at rest when $t = 0$, $t = 50$ and $t = 100$. [2]

(ii) Find the values of v at the times for which the acceleration of P is zero, and sketch the velocity-time graph for P's motion for $0 \leqslant t \leqslant 100$. [7]

(iii) Find the greatest distance of P from O for $0 \leqslant t \leqslant 100$. [4]

Answer: (ii) The values of v when $a = 0$ are 4.81 and –4.81 (iii) Greatest distance of P from O is 156 m   43/J15/7

2 Two particles A and B start to move at the same instant from a point O. The particles move in the same direction along the same straight line. The acceleration of A at time t s after starting to move is $a\,\text{m s}^{-2}$, where $a = 0.05 - 0.0002t$.

(i) Find A's velocity when $t = 200$ and when $t = 500$. [4]

B moves with constant acceleration for the first 200 s and has the same velocity as A when $t = 200$. B moves with constant retardation from $t = 200$ to $t = 500$ and has the same velocity as A when $t = 500$.

(ii) Find the distance between A and B when $t = 500$. [6]

Answer: (i) Velocity of A when t =200 is 6 ms^{-1} Velocity of A when t = 500 is 0 ms^{-1} 41/J15/6
(ii) Distance between A and B when t = 500 is 2083 m −1500 m = 583 m

3 A particle P starts from rest and moves in a straight line for 18 seconds. For the first 8 seconds of the motion P has constant acceleration $0.25\,\text{m s}^{-2}$. Subsequently P's velocity, $v\,\text{m s}^{-1}$ at time t seconds after the motion started, is given by

$$v = -0.1t^2 + 2.4t - k,$$

where $8 \leqslant t \leqslant 18$ and k is a constant.

(i) Find the value of v when $t = 8$ and hence find the value of k. [2]

(ii) Find the maximum velocity of P. [2]

(iii) Find the displacement of P from its initial position when $t = 18$. [3]

Answer: 2, 10.8, 3.6, 34.7   43/N14/4

4 Particles P and Q move on a straight line AOB. The particles leave O simultaneously, with P moving towards A and with Q moving towards B. The initial speed of P is $1.3\,\text{m s}^{-1}$ and its acceleration in the direction OA is $0.1\,\text{m s}^{-2}$. Q moves with acceleration in the direction OB of $0.016t\,\text{m s}^{-2}$, where t seconds is the time elapsed since the instant that P and Q started to move from O. When $t = 20$, particle P passes through A and particle Q passes through B.

(i) Given that the speed of Q at B is the same as the speed of P at A, find the speed of Q at time $t = 0$. [4]

(ii) Find the distance AB. [3]

Answer: 0.1, 69.3 41/N14/4

5 A particle of mass 3 kg falls from rest at a point 5 m above the surface of a liquid which is in a container. There is no instantaneous change in speed of the particle as it enters the liquid. The depth of the liquid in the container is 4 m. The downward acceleration of the particle while it is moving in the liquid is $5.5\,\text{m}\,\text{s}^{-2}$.

(i) Find the resistance to motion of the particle while it is moving in the liquid. [2]

(ii) Sketch the velocity-time graph for the motion of the particle, from the time it starts to move until the time it reaches the bottom of the container. Show on your sketch the velocity and the time when the particle enters the liquid, and when the particle reaches the bottom of the container. [7]

Answer: 13.5 41/N14/6

6 A small ball of mass 0.4 kg is released from rest at a point 5 m above horizontal ground. At the instant the ball hits the ground it loses 12.8 J of kinetic energy and starts to move upwards.

(i) Show that the greatest height above the ground that the ball reaches after hitting the ground is 1.8 m. [4]

(ii) Find the time taken for the ball's motion from its release until reaching this greatest height. [3]

Answer: 1.6 s 43/J14/4

7 A particle starts from rest at a point O and moves in a horizontal straight line. The velocity of the particle is $v\,\text{m}\,\text{s}^{-1}$ at time t s after leaving O. For $0 \leqslant t < 60$, the velocity is given by

$$v = 0.05t - 0.0005t^2.$$

The particle hits a wall at the instant when $t = 60$, and reverses the direction of its motion. The particle subsequently comes to rest at the point A when $t = 100$, and for $60 < t \leqslant 100$ the velocity is given by

$$v = 0.025t - 2.5.$$

(i) Find the velocity of the particle immediately before it hits the wall, and its velocity immediately after its hits the wall. [2]

(ii) Find the total distance travelled by the particle. [4]

(iii) Find the maximum speed of the particle and sketch the particle's velocity-time graph for $0 \leqslant t \leqslant 100$, showing the value of t for which the speed is greatest. [4]

Answer: 1.2 ms^{-1} $^{-}$1 ms^{-1} 74 m 1.25 ms^{-1} 43/J14/6

8 A particle is projected vertically upwards with speed $9\,\mathrm{m\,s^{-1}}$ from a point 3.15 m above horizontal ground. The particle moves freely under gravity until it hits the ground. For the particle's motion from the instant of projection until the particle hits the ground, find the total distance travelled and the total time taken. [6]

Answer: Total distance travelled = 11.25 m. Total time taken = 2.1 s 41/J14/4

9 Two cyclists P and Q travel along a straight road ABC, starting simultaneously at A and arriving simultaneously at C. Both cyclists pass through B 400 s after leaving A. Cyclist P starts with speed $3\,\mathrm{m\,s^{-1}}$ and increases this speed with constant acceleration $0.005\,\mathrm{m\,s^{-2}}$ until he reaches B.

(i) Show that the distance AB is 1600 m and find P's speed at B. [3]

Cyclist Q travels from A to B with speed $v\,\mathrm{m\,s^{-1}}$ at time t seconds after leaving A, where

$$v = 0.04t - 0.0001t^2 + k,$$

and k is a constant.

(ii) Find the value of k and the maximum speed of Q before he has reached B. [6]

Cyclist P travels from B to C, a distance of 1400 m, at the speed he had reached at B. Cyclist Q travels from B to C with constant acceleration $a\,\mathrm{m\,s^{-2}}$.

(iii) Find the time taken for the cyclists to travel from B to C and find the value of a. [4]

Answer: Speed of $P = 5\,\mathrm{ms^{-1}}$ 41/J14/7

10 A car travels in a straight line from A to B, a distance of 12 km, taking 552 seconds. The car starts from rest at A and accelerates for T_1 s at $0.3\,\mathrm{m\,s^{-2}}$, reaching a speed of $V\,\mathrm{m\,s^{-1}}$. The car then continues to move at $V\,\mathrm{m\,s^{-1}}$ for T_2 s. It then decelerates for T_3 s at $1\,\mathrm{m\,s^{-2}}$, coming to rest at B.

(i) Sketch the velocity-time graph for the motion and express T_1 and T_3 in terms of V. [3]

(ii) Express the total distance travelled in terms of V and show that $13V^2 - 3312V + 72\,000 = 0$. Hence find the value of V. [5]

Answer: $T_1 = V \div 0.3$ $T_3 = V$ 24 43/N13/5

11 A vehicle starts from rest at a point O and moves in a straight line. Its speed $v\,\mathrm{m\,s^{-1}}$ at time t seconds after leaving O is defined as follows.

$$\text{For } 0 \leqslant t \leqslant 60, \quad v = k_1 t - 0.005t^2,$$
$$\text{for } t \geqslant 60, \quad v = \frac{k_2}{\sqrt{t}}.$$

The distance travelled by the vehicle during the first 60 s is 540 m.

(i) Find the value of the constant k_1 and show that $k_2 = 12\sqrt{(60)}$. [5]

(ii) Find an expression in terms of t for the total distance travelled when $t \geqslant 60$. [2]

(iii) Find the speed of the vehicle when it has travelled a total distance of 1260 m. [3]

Answer. 0.5 $24\sqrt{(60t)} - 900$ $8\ ms^{-1}$ 43/N13/7

12 A particle P starts from rest at a point O and moves in a straight line. P has acceleration $0.6t\ m\,s^{-2}$ at time t seconds after leaving O, until $t = 10$.

(i) Find the velocity and displacement from O of P when $t = 10$. [5]

After $t = 10$, P has acceleration $-0.4t\ m\,s^{-2}$ until it comes to rest at a point A.

(ii) Find the distance OA. [7]

Answer: **(i)** Velocity = 30 ms^{-1}, displacement = 100 m **(ii)** 194 m 41/N13/7

13 An aeroplane moves along a straight horizontal runway before taking off. It starts from rest at O and has speed $90\ m\,s^{-1}$ at the instant it takes off. While the aeroplane is on the runway at time t seconds after leaving O, its acceleration is $(1.5 + 0.012t)\ m\,s^{-2}$. Find

(i) the value of t at the instant the aeroplane takes off, [4]

(ii) the distance travelled by the aeroplane on the runway. [3]

Answer. 50; 2125 m 43/J13/4

14 A particle P is projected vertically upwards from a point on the ground with speed $17\ m\,s^{-1}$. Another particle Q is projected vertically upwards from the same point with speed $7\ m\,s^{-1}$. Particle Q is projected T seconds later than particle P.

(i) Given that the particles reach the ground at the same instant, find the value of T. [2]

(ii) At a certain instant when both P and Q are in motion, P is 5 m higher than Q. Find the magnitude and direction of the velocity of each of the particles at this instant. [6]

Answer. 2; 12 ms^{-1} and 2 ms^{-1} downwards 43/J13/5

15 The top of a cliff is 40 metres above the level of the sea. A man in a boat, close to the bottom of the cliff, is in difficulty and fires a distress signal vertically upwards from sea level. Find

(i) the speed of projection of the signal given that it reaches a height of 5 m above the top of the cliff, [2]

(ii) the length of time for which the signal is above the level of the top of the cliff. [2]

The man fires another distress signal vertically upwards from sea level. This signal is above the level of the top of the cliff for $\sqrt{(17)}$ s.

(iii) Find the speed of projection of the second signal. [3]

Answer: i) 30 ii) 2 iii) 35 41/J13/3

16 A car driver makes a journey in a straight line from A to B, starting from rest. The speed of the car increases to a maximum, then decreases until the car is at rest at B. The distance travelled by the car t seconds after leaving A is $0.000\,011\,7(400t^3 - 3t^4)$ metres.

(i) Find the distance AB. [3]

(ii) Find the maximum speed of the car. [4]

(iii) Find the acceleration of the car

(a) as it starts from A,

(b) as it arrives at B.

[2]

(iv) Sketch the velocity-time graph for the journey. [2]

Answer: i) 1170 ii) 20.8 iii) -1.40 iv) 41/J13/7

17 A particle moves in a straight line. Its velocity t seconds after leaving a fixed point O on the line is $v\,\text{m s}^{-1}$, where $v = 0.2t + 0.006t^2$. For the instant when the acceleration of the particle is 2.5 times its initial acceleration,

(i) show that $t = 25$, [3]

(ii) find the displacement of the particle from O. [3]

Answer. (ii) 93.8 m. 43/N12/2

18 A particle P is projected vertically upwards, from a point O, with a velocity of $8\,\text{m s}^{-1}$. The point A is the highest point reached by P. Find

(i) the speed of P when it is at the mid-point of OA, [4]

(ii) the time taken for P to reach the mid-point of OA while moving upwards. [2]

Answers: (i) 5.66 ms^{-1}; (ii) 0.234 s. 43/N12/3

19 An object is released from rest at a height of 125 m above horizontal ground and falls freely under gravity, hitting a moving target P. The target P is moving on the ground in a straight line, with constant acceleration $0.8\,\text{m s}^{-2}$. At the instant the object is released P passes through a point O with speed $5\,\text{m s}^{-1}$. Find the distance from O to the point where P is hit by the object. [4]

Answer: 35 m. 41/N12/1

20 Particle P travels along a straight line from A to B with constant acceleration $0.05\,\text{m s}^{-2}$. Its speed at A is $2\,\text{m s}^{-1}$ and its speed at B is $5\,\text{m s}^{-1}$.

(i) Find the time taken for P to travel from A to B, and find also the distance AB. [3]

Particle Q also travels along the same straight line from A to B, starting from rest at A. At time t s after leaving A, the speed of Q is $kt^3\,\text{m s}^{-1}$, where k is a constant. Q takes the same time to travel from A to B as P does.

(ii) Find the value of k and find Q's speed at B. [5]

Answers: (i) 60 s, 210 m; (ii) $k = 0.0000648$, 14 ms^{-1}. 41/N12/5

21 A particle P travels from a point O along a straight line and comes to instantaneous rest at a point A. The velocity of P at time t s after leaving O is v m s^{-1}, where $v = 0.027(10t^2 - t^3)$. Find

(i) the distance OA, [4]

(ii) the maximum velocity of P while moving from O to A. [3]

Answers: (i) 22.5 m (ii) 4 ms^{-1} 43/J12/3

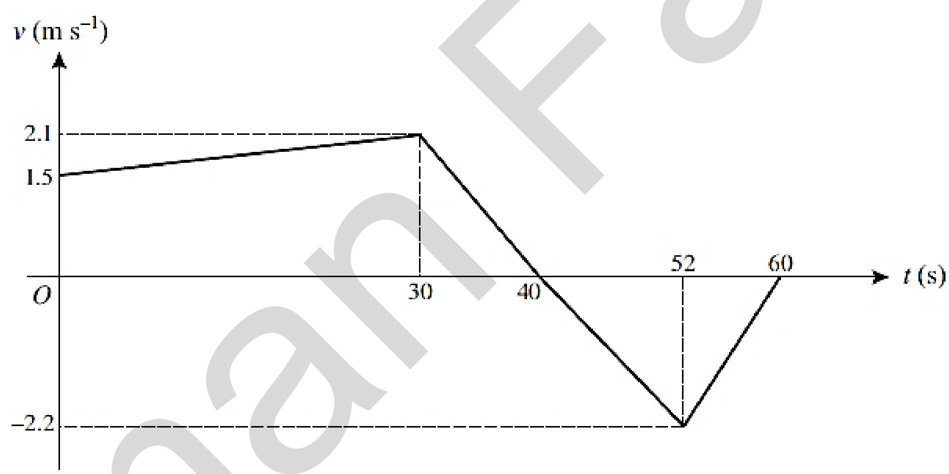
22 A particle P starts at the point O and travels in a straight line. At time t seconds after leaving O the velocity of P is v m s^{-1}, where $v = 0.75t^2 - 0.0625t^3$. Find

(i) the positive value of t for which the acceleration is zero, [3]

(ii) the distance travelled by P before it changes its direction of motion. [5]

Answers: (i) $t = 8$ s (ii) 108 m 41/J12/4

23



A woman walks in a straight line. The woman's velocity t seconds after passing through a fixed point A on the line is v m s^{-1}. The graph of v against t consists of 4 straight line segments (see diagram). The woman is at the point B when $t = 60$. Find

(i) the woman's acceleration for $0 < t < 30$ and for $30 < t < 40$, [3]

(ii) the distance AB, [2]

(iii) the total distance walked by the woman. [1]

Answers: (i) 0.02 ms^{-2}, –0.21 ms^{-2}; (ii) 42.5 m; (iii) 86.5 m 43/N11/1

24 A particle P moves in a straight line. It starts from rest at A and comes to rest instantaneously at B. The velocity of P at time t seconds after leaving A is $v\,\text{m s}^{-1}$, where $v = 6t^2 - kt^3$ and k is a constant.

(i) Find an expression for the displacement of P from A in terms of t and k. [2]

(ii) Find an expression for t in terms of k when P is at B. [1]

Given that the distance AB is 108 m, find

(iii) the value of k, [2]

(iv) the maximum value of v when the particle is moving from A towards B. [3]

Answers: (i) $2t^3 - kt^4/4$; (ii) $6/k$; (iii) 1; (iv) 32 43/N11/5

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25 A particle P starts from a point O and moves along a straight line. P's velocity t s after leaving O is $v\,\text{m s}^{-1}$, where

$$v = 0.16t^{\frac{3}{2}} - 0.016t^2.$$

P comes to rest instantaneously at the point A.

(i) Verify that the value of t when P is at A is 100. [1]

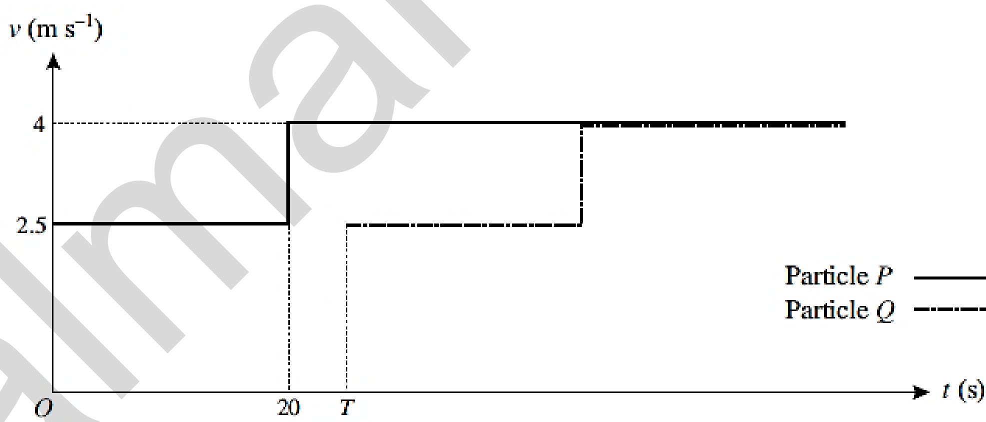
(ii) Find the maximum speed of P in the interval $0 < t < 100$. [4]

(iii) Find the distance OA. [3]

(iv) Find the value of t when P passes through O on returning from A. [2]

Answers: (ii) 16.9 ms^{-1} (or 16 7/8 ms^{-1}); (iii) 1070 m; (iv) t = 144 seconds 41/N11/7

26



The diagram shows the velocity-time graphs for the motion of two particles P and Q, which travel in the same direction along a straight line. P and Q both start at the same point X on the line, but Q starts to move T s later than P. Each particle moves with speed $2.5\,\text{m s}^{-1}$ for the first 20 s of its motion. The speed of each particle changes instantaneously to $4\,\text{m s}^{-1}$ after it has been moving for 20 s and the particle continues at this speed.

(i) Make a rough copy of the diagram and shade the region whose area represents the displacement of P from X at the instant when Q starts. [1]

It is given that P has travelled 70 m at the instant when Q starts.

(ii) Find the value of T. [2]

(iii) Find the distance between P and Q when Q's speed reaches 4 m s^{-1}. [2]

(iv) Sketch a single diagram showing the displacement-time graphs for both P and Q, with values shown on the t-axis at which the speed of either particle changes. [2]

Answers: (ii) 25; (iii) 100 m. 43/J11/4

27 A particle travels in a straight line from A to B in 20 s. Its acceleration t seconds after leaving A is $a\text{ m s}^{-2}$, where $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$. It is given that the particle comes to rest at B.

(i) Show that the initial speed of the particle is zero. [4]

(ii) Find the maximum speed of the particle. [2]

(iii) Find the distance AB. [4]

Answers: (ii) 5.27 ms^{-1}; (iii) 50 m. 43/J11/7

28 A train starts from rest at a station A and travels in a straight line to station B, where it comes to rest. The train moves with constant acceleration 0.025 m s^{-2} for the first 600 s, with constant speed for the next 2600 s, and finally with constant deceleration 0.0375 m s^{-2}.

(i) Find the total time taken for the train to travel from A to B. [4]

(ii) Sketch the velocity-time graph for the journey and find the distance AB. [3]

(iii) The speed of the train t seconds after leaving A is 7.5 m s^{-1}. State the possible values of t. [1]

Answers: (i) 3600 s; (ii) 46 500 m (or 46.5 km); (iii) 300, 3400. 41/J11/5

29 A particle travels in a straight line from a point P to a point Q. Its velocity t seconds after leaving P is $v\text{ m s}^{-1}$, where $v = 4t - \frac{1}{16}t^3$. The distance PQ is 64 m.

(i) Find the time taken for the particle to travel from P to Q. [5]

(ii) Find the set of values of t for which the acceleration of the particle is positive. [4]

Answers: (i) 8 s; (ii) $0 < t < 4.62$. 41/J11/6

30 A particle starts from rest at a point X and moves in a straight line until, 60 seconds later, it reaches a point Y. At time t s after leaving X, the acceleration of the particle is

$$\begin{aligned} 0.75\text{ m s}^{-2} &\quad \text{for} \quad 0 < t < 4,\\ 0\text{ m s}^{-2} &\quad \text{for} \quad 4 < t < 54,\\ -0.5\text{ m s}^{-2} &\quad \text{for} \quad 54 < t < 60. \end{aligned}$$

(i) Find the velocity of the particle when $t = 4$ and when $t = 60$, and sketch the velocity-time graph. [5]

(ii) Find the distance XY. [2]

Answers: (i) 3 ms^{-1}, 0 ms^{-1}; (ii) 165 m. 43/N10/4

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31 A particle travels along a straight line. It starts from rest at a point A on the line and comes to rest again, 10 seconds later, at another point B on the line. The velocity t seconds after leaving A is

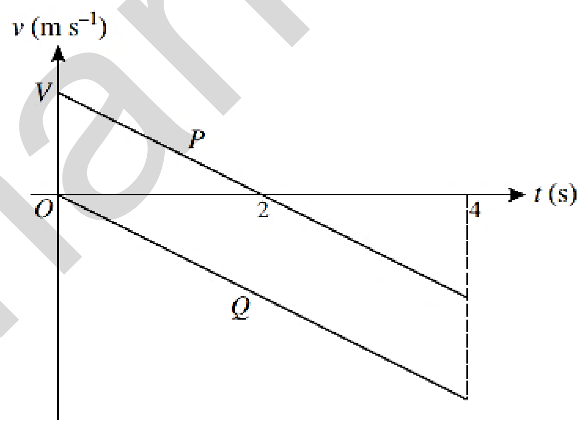
$$\begin{aligned} 0.72t^2 - 0.096t^3 &\quad \text{for} \quad 0 \leqslant t \leqslant 5,\\ 2.4t - 0.24t^2 &\quad \text{for} \quad 5 \leqslant t \leqslant 10. \end{aligned}$$

(i) Show that there is no instantaneous change in the acceleration of the particle when $t = 5$. [4]

(ii) Find the distance AB. [4]

Answer: (ii) 35 m. 43/N10/6

32



Two particles P and Q move vertically under gravity. The graphs show the upward velocity v m s^{-1} of the particles at time t s, for $0 \leqslant t \leqslant 4$. P starts with velocity V m s^{-1} and Q starts from rest.

(i) Find the value of V. [2]

Given that Q reaches the horizontal ground when $t = 4$, find

(ii) the speed with which Q reaches the ground, [1]

(iii) the height of Q above the ground when $t = 0$. [2]

Answer: i) 20 ii) 40 iii) 80 41/N10/1

33 A particle P starts from a fixed point O at time $t = 0$, where t is in seconds, and moves with constant acceleration in a straight line. The initial velocity of P is $1.5\,\text{m s}^{-1}$ and its velocity when $t = 10$ is $3.5\,\text{m s}^{-1}$.

(i) Find the displacement of P from O when $t = 10$. [2]

Another particle Q also starts from O when $t = 0$ and moves along the same straight line as P. The acceleration of Q at time t is $0.03t\,\text{m s}^{-2}$.

(ii) Given that Q has the same velocity as P when $t = 10$, show that it also has the same displacement from O as P when $t = 10$. [5]

Answer: i) 25 ii) 41/N10/4

34 A particle starts at a point O and moves along a straight line. Its velocity t s after leaving O is $(1.2t - 0.12t^2)\,\text{m s}^{-1}$. Find the displacement of the particle from O when its acceleration is $0.6\,\text{m s}^{-2}$. [5]

Answer: 3.13 m. 43/J10/2

35 A ball moves on the horizontal surface of a billiards table with deceleration of constant magnitude $d\,\text{m s}^{-2}$. The ball starts at A with speed $1.4\,\text{m s}^{-1}$ and reaches the edge of the table at B, 1.2 s later, with speed $1.1\,\text{m s}^{-1}$.

(i) Find the distance AB and the value of d. [3]

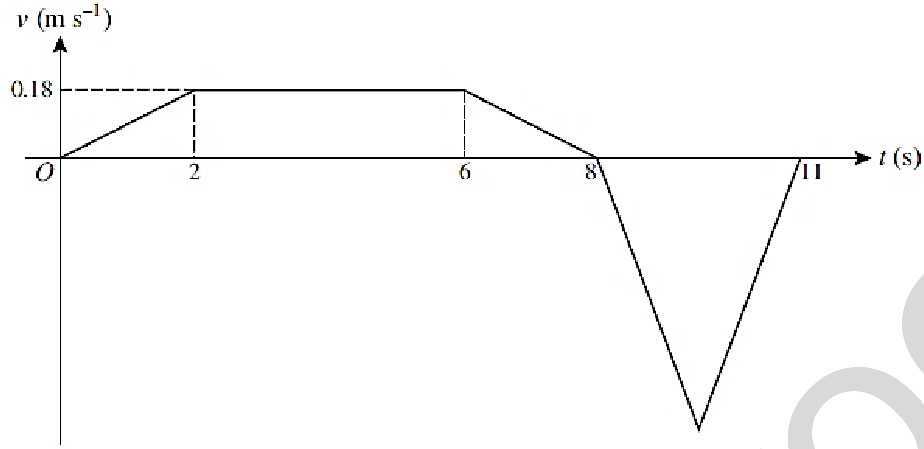
AB is at right angles to the edge of the table containing B. The table has a low wall along each of its edges and the ball rebounds from the wall at B and moves directly towards A. The ball comes to rest at C where the distance BC is 2 m.

(ii) Find the speed with which the ball starts to move towards A and the time taken for the ball to travel from B to C. [3]

(iii) Sketch a velocity-time graph for the motion of the ball, from the time the ball leaves A until it comes to rest at C, showing on the axes the values of the velocity and the time when the ball is at A, at B and at C. [2]

Answers: (i) 1.5 m, 0.25; (ii) $1\,\text{ms}^{-1}$, 4 s. 43/J10/5

36



The diagram shows the velocity-time graph for the motion of a machine's cutting tool. The graph consists of five straight line segments. The tool moves forward for 8 s while cutting and then takes 3 s to return to its starting position. Find

(i) the acceleration of the tool during the first 2 s of the motion, [1]

(ii) the distance the tool moves forward while cutting, [2]

(iii) the greatest speed of the tool during the return to its starting position. [2]

Answer: i) 0.09 ii) 1.08 iii) 0.72 41/J10/2

37 A vehicle is moving in a straight line. The velocity $v\,\text{m s}^{-1}$ at time t s after the vehicle starts is given by

$$v = A(t - 0.05t^2) \quad \text{for } 0 \leqslant t \leqslant 15,$$

$$v = \frac{B}{t^2} \quad \text{for } t \geqslant 15,$$

where A and B are constants. The distance travelled by the vehicle between $t = 0$ and $t = 15$ is 225 m.

(i) Find the value of A and show that $B = 3375$. [5]

(ii) Find an expression in terms of t for the total distance travelled by the vehicle when $t \geqslant 15$. [3]

(iii) Find the speed of the vehicle when it has travelled a total distance of 315 m. [3]

Answer: i) 4 ii) 450-3375/t iii) 5.4 41/J10/7

38 A lift moves upwards from rest and accelerates at $0.9\,\text{m s}^{-2}$ for 3 s. The lift then travels for 6 s at constant speed and finally slows down, with a constant deceleration, stopping in a further 4 s.

(i) Sketch a velocity-time graph for the motion. [3]

(ii) Find the total distance travelled by the lift. [2]

Answer: Trapezium with 0, 3, 9 and 13 shown on the t-axis. 2.7 shown on the v-axis J16/41/Q1

Answer: Total distance travelled = 25.7 m

39 A particle P moves in a straight line. It starts at a point O on the line and at time t s after leaving O it has a velocity $v\,\text{m s}^{-1}$, where $v = 6t^2 - 30t + 24$.

(i) Find the set of values of t for which the acceleration of the particle is negative. [2]

(ii) Find the distance between the two positions at which P is at instantaneous rest. [4]

(iii) Find the two positive values of t at which P passes through O. [3]

Answer: $t < 2.5$ *Answer*: Distance between the two positions = 27 m J16/41/Q6

Answer: The two values of t at which the particle passes through O are $t = 2.31$ and $t = 5.19$

40 Alan starts walking from a point O, at a constant speed of $4\,\text{m s}^{-1}$, along a horizontal path. Ben walks along the same path, also starting from O. Ben starts from rest 5 s after Alan and accelerates at $1.2\,\text{m s}^{-2}$ for 5 s. Ben then continues to walk at a constant speed until he is at the same point, P, as Alan.

(i) Find how far Ben has travelled when he has been walking for 5 s and find his speed at this instant. [2]

(ii) Find the distance OP. [3]

Answer: 15 m 6 ms^{-1} 90 m J16/43/Q2

41 A particle P moves in a straight line $ABCD$ with constant deceleration. The velocities of P at A, B and C are $20\,\text{m s}^{-1}$, $12\,\text{m s}^{-1}$ and $6\,\text{m s}^{-1}$ respectively.

(i) Find the ratio of distances $AB : BC$. [4]

(ii) The particle comes to rest at D. Given that the distance AD is 80 m, find the distance BC. [3]

Answers: **(i)** $AB : BC = 64 : 27$ **(ii)** The distance BC is 21.6 m J17/41/Q5

42 A particle P moves in a straight line passing through a point O. At time t s, the velocity of P, $v\,\text{m s}^{-1}$, is given by $v = qt + rt^2$, where q and r are constants. The particle has velocity $4\,\text{m s}^{-1}$ when $t = 1$ and when $t = 2$.

(i) Show that, when $t = 0.5$, the acceleration of P is $4\,\text{m s}^{-2}$. [4]

(ii) Find the values of t when P is at instantaneous rest. [2]

(iii) The particle is at O when $t = 3$. Find the distance of P from O when $t = 0$. [4]

Answers: **(i)** Acceleration at $t = 0.5$ is 4 ms^{-2} (Answer given) **(ii)** The values of t when P is at instantaneous rest are $t = 0$ and $t = 3$ **(iii)** The distance of P from O when $t = 0$ is 9 m J17/41/Q6

43 A train travels between two stations, A and B. The train starts from rest at A and accelerates at a constant rate for T s until it reaches a speed of $25\,\text{m s}^{-1}$. It then travels at this constant speed before decelerating at a constant rate, coming to rest at B. The magnitude of the train's deceleration is twice the magnitude of its acceleration. The total time taken for the journey is 180 s.

(i) Sketch the velocity-time graph for the train's journey from A to B. [1]

(ii) Find an expression, in terms of T, for the length of time for which the train is travelling with constant speed. [2]

(iii) The distance from A to B is 3300 m. Find how far the train travels while it is decelerating. [3]

Answers: **(ii)** $180 - 1.5T$ **(iii)** 400 m J17/43/Q3

44 A particle P moves in a straight line starting from a point O. At time t s after leaving O, the velocity, $v\,\text{m s}^{-1}$, of P is given by $v = (2t - 5)^3$.

(i) Find the values of t when the acceleration of P is $54\,\text{m s}^{-2}$. [3]

(ii) Find an expression for the displacement of P from O at time t s. [3]

Answers: **(i)** $t = 1$ or 4 **(ii)** $s = (2t - 5)^4/8 - 625/8$ J17/43/Q4

45 A particle is projected vertically upwards from a point O with a speed of $12\,\text{m s}^{-1}$. Two seconds later a second particle is projected vertically upwards from O with a speed of $20\,\text{m s}^{-1}$. At time t s after the second particle is projected, the two particles collide.

(i) Find t. [5]

(ii) Hence find the height above O at which the particles collide. [1]

Answers: **(i)** 0.143 **(ii)** 2.76 m J17/43/Q5

46 A particle P is projected vertically upwards with speed $24\,\text{m s}^{-1}$ from a point 5 m above ground level. Find the time from projection until P reaches the ground. [3]

Answer: Time from projection until P reaches the ground is 5 seconds J18/41/Q1

47 A particle P moves in a straight line starting from a point O. At time t s after leaving O, the displacement s m from O is given by $s = t^3 - 4t^2 + 4t$ and the velocity is v m s^{-1}.

(i) Find an expression for v in terms of t. [2]

(ii) Find the two values of t for which P is at instantaneous rest. [2]

(iii) Find the minimum velocity of P. [3]

Answers: (i) The velocity is given by $v = 3t^2 - 8t + 4$ (ii) The two values of t at which P is at instantaneous rest are $t = 2$ and $t = \frac{2}{3}$ (iii) The minimum velocity of P occurs when $t = \frac{4}{3}$ and is $v = -\frac{4}{3}$ ms^{-1} J18/41/Q4

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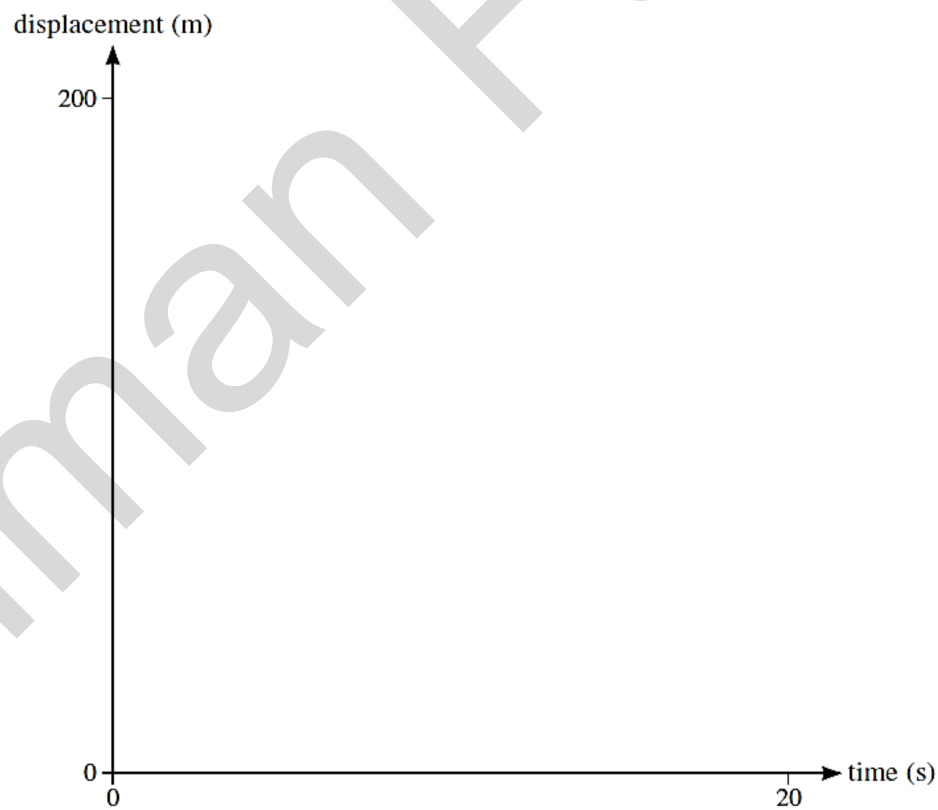
48 A sprinter runs a race of 200 m. His total time for running the race is 20 s. He starts from rest and accelerates uniformly for 6 s, reaching a speed of 12 m s^{-1}. He maintains this speed for the next 10 s, before decelerating uniformly to cross the finishing line with speed V m s^{-1}.

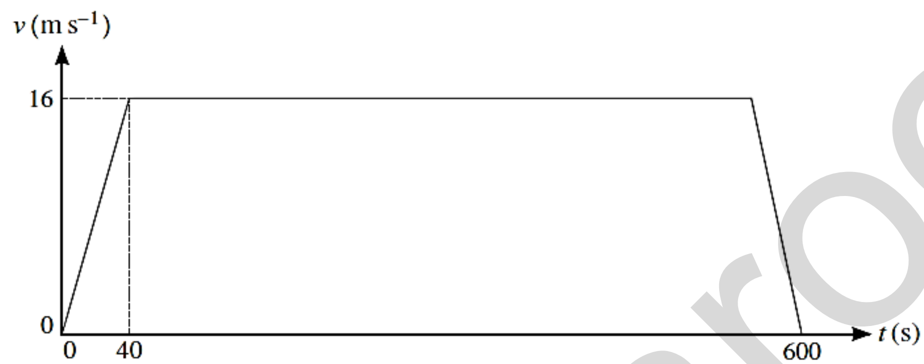
(i) Find the distance travelled by the sprinter in the first 16 s of the race. Hence sketch a displacement-time graph for the 20 s of the sprinter's race. [6]



(ii) Find the value of V. [2]

Answers: (i) The distance travelled in the first 16 seconds is 156 m J18/41/Q5
The displacement time graph involves three curves which join smoothly.
A curve, concave upwards from (0,0) to (6,36).
A straight line from (6,36) to (16,156).
A curve, concave downwards from (16,156) to (20,200)
(ii) The value of V is 10

49



The diagram shows the velocity-time graph for a train which travels from rest at one station to rest at the next station. The graph consists of three straight line segments. The distance between the two stations is 9040 m.

(i) Find the acceleration of the train during the first 40 s. [1]

(ii) Find the length of time for which the train is travelling at constant speed. [2]

(iii) Find the distance travelled by the train while it is decelerating. [2]

Answers: (i) $0.4\ \text{ms}^{-2}$ (ii) 530 s (iii) 240 m J18/43/Q1

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50 A small ball is projected vertically downwards with speed $5\ \text{m s}^{-1}$ from a point A at a height of 7.2 m above horizontal ground. The ball hits the ground with speed $V\ \text{m s}^{-1}$ and rebounds vertically upwards with speed $\frac{1}{2}V\ \text{m s}^{-1}$. The highest point the ball reaches after rebounding is B. Find V and hence find the total time taken for the ball to reach the ground from A and rebound to B. [5]

Answer: 13, 1.45 s J18/43/Q2

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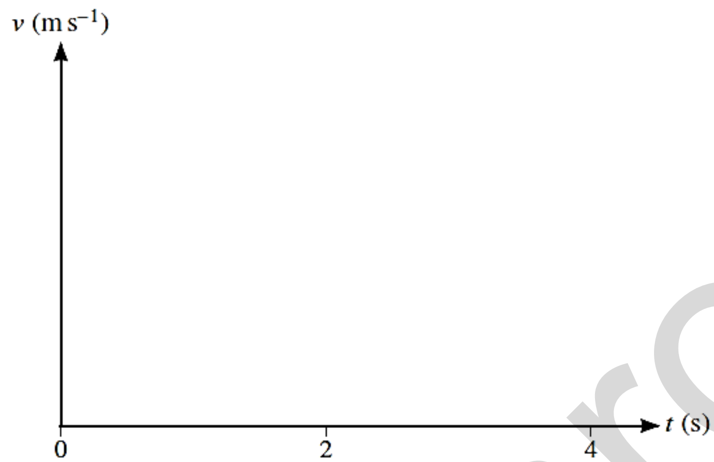
51 A particle P moves in a straight line starting from a point O. The velocity $v\ \text{m s}^{-1}$ of P at time t s is given by

$$v = 12t - 4t^2 \quad \text{for } 0 \leqslant t \leqslant 2,$$
$$v = 16 - 4t \quad \text{for } 2 \leqslant t \leqslant 4.$$

(i) Find the maximum velocity of P during the first 2 s. [3]

(ii) Determine, with justification, whether there is any instantaneous change in the acceleration of P when $t = 2$. [2]

(iii) Sketch the velocity-time graph for $0 \leqslant t \leqslant 4$. [3]



(iv) Find the distance travelled by P in the interval $0 \leqslant t \leqslant 4$. [5]

Answers: (i) $9\,\text{ms}^{-1}$ (ii) No instantaneous change (iv) 21.3 m — J18/43/Q7

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52 A particle P is projected vertically upwards from a point O. When the particle is at a height of 0.5 m, its speed is $6\,\text{m}\,\text{s}^{-1}$. Find

(i) the greatest height reached by the particle above O, [3]

(ii) the time after projection at which the particle returns to O. [3]

N16/41/Q3

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53 A racing car is moving in a straight line. The acceleration $a\,\text{m}\,\text{s}^{-2}$ at time t s after the car starts from rest is given by

$$a = 15t - 3t^2 \qquad \text{for } 0 \leqslant t \leqslant 5,$$

$$a = -\frac{625}{t^2} \qquad \text{for } 5 < t \leqslant k,$$

where k is a constant.

(i) Find the maximum acceleration of the car in the first five seconds of its motion. [3]

(ii) Find the distance of the car from its starting point when $t = 5$. [3]

(iii) The car comes to rest when $t = k$. Find the value of k. [5]

Answer: The maximum acceleration in the first five seconds is $18.75\,\text{ms}^{-2}$ — N16/41/Q7

Answer: The distance of the car from its starting point at $t = 5$ is 156.25 m

Answer: $k = 10$

54 A ball A is released from rest at the top of a tall tower. One second later, another ball B is projected vertically upwards from ground level near the bottom of the tower with a speed of $20\,\text{m s}^{-1}$. The two balls are at the same height 1.5 s after ball B is projected.

(i) Show that the height of the tower is 50 m. [3]

(ii) Find the length of time for which ball B has been in motion when ball A reaches the ground. Hence find the total distance travelled by ball B up to the instant when ball A reaches the ground. [5]

Answer: Length of time for which ball B is in motion when ball A reaches the ground is 2.16 s
Total distance travelled by ball B up to the instant ball A reaches the ground is 20.1 m N16/43/Q4

55 A particle P starts from a fixed point O and moves in a straight line. At time t s after leaving O, the velocity $v\,\text{m s}^{-1}$ of P is given by $v = 6t - 0.3t^2$. The particle comes to instantaneous rest at point X.

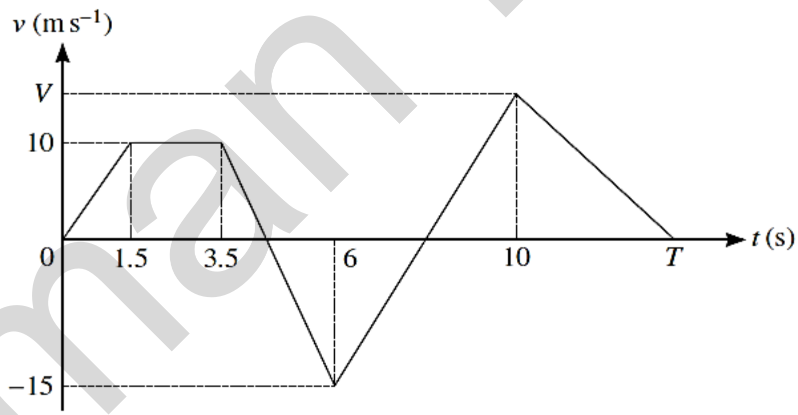
(i) Find the distance OX. [4]

A second particle Q starts from rest from O, at the same instant as P, and also travels in a straight line. The acceleration $a\,\text{m s}^{-2}$ of Q is given by $a = k - 12t$, where k is a constant. The displacement of Q from O is 400 m when $t = 10$.

(ii) Find the value of k. [4]

Answer: The distance OX is 400 m *Answer:* k = 48 N16/43/Q5

56



The diagram shows the velocity-time graph of a particle which moves in a straight line. The graph consists of 5 straight line segments. The particle starts from rest at a point A at time $t = 0$, and initially travels towards point B on the line.

(i) Show that the acceleration of the particle between $t = 3.5$ and $t = 6$ is $-10\,\text{m s}^{-2}$. [1]

(ii) The acceleration of the particle between $t = 6$ and $t = 10$ is $7.5\,\text{m s}^{-2}$. When $t = 10$ the velocity of the particle is $V\,\text{m s}^{-1}$. Find the value of V. [2]

(iii) The particle comes to rest at B at time T s. Given that the total distance travelled by the particle between $t = 0$ and $t = T$ is 100 m, find the value of T. [4]

Answer: The acceleration between $t = 3.5$ and $t = 6$ is $-10\ \mathrm{m\,s^{-2}}$ (given) *Answer:* $V = 15$ N17/41/Q4

Answer: $T = 13.5$

---

57 A particle starts from a point O and moves in a straight line. The velocity of the particle at time t s after leaving O is $v\ \mathrm{m\,s^{-1}}$, where

$$v = 1.5 + 0.4t \qquad \text{for } 0 \leqslant t \leqslant 5,$$
$$v = \frac{100}{t^2} - 0.1t \qquad \text{for } t \geqslant 5.$$

(i) Find the acceleration of the particle during the first 5 seconds of motion. [1]

(ii) Find the value of t when the particle is instantaneously at rest. [2]

(iii) Find the total distance travelled by the particle in the first 10 seconds of motion. [5]

Answer: Acceleration is $0.4\ \mathrm{m\,s^{-2}}$ *Answer:* $t = 10$ N17/41/Q5

Answer: The distance travelled in the first ten seconds of motion is 18.75 m

---

58 A particle starts from a fixed origin with velocity $0.4\ \mathrm{m\,s^{-1}}$ and moves in a straight line. The acceleration $a\ \mathrm{m\,s^{-2}}$ of the particle t s after it leaves the origin is given by $a = k(3t^2 - 12t + 2)$, where k is a constant. When $t = 1$, the velocity of P is $0.1\ \mathrm{m\,s^{-1}}$.

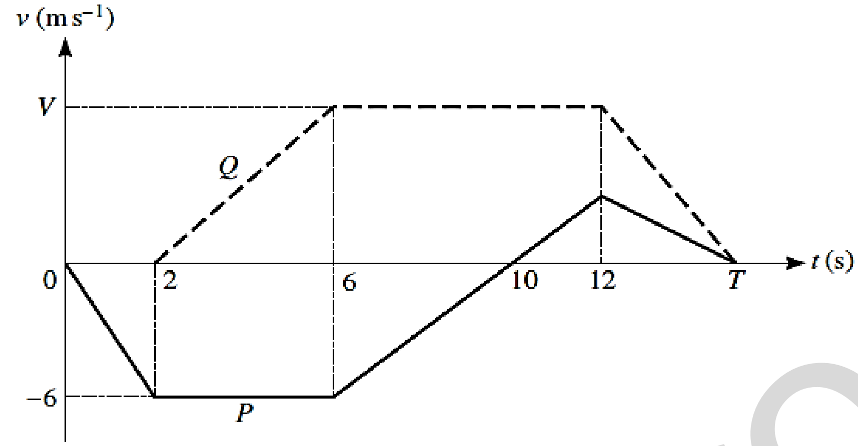
(i) Show that the value of k is 0.1. **[5]**

(ii) Find an expression for the displacement of the particle from the origin in terms of t. [2]

(iii) Hence verify that the particle is again at the origin at $t = 2$. [1]

Answers: **(i)** Answer Given **(ii)** $s = 0.025t^4 - 0.2t^3 + 0.1t^2 + 0.4t$ **(iii)** Answer Given N17/43/Q5

59



The diagram shows the velocity-time graphs for two particles, P and Q, which are moving in the same straight line. The graph for P consists of four straight line segments. The graph for Q consists of three straight line segments. Both particles start from the same initial position O on the line. Q starts 2 seconds after P and both particles come to rest at time $t = T$. The greatest velocity of Q is V m s^{-1}.

(i) Find the displacement of P from O at $t = 10$. [1]

(ii) Find the velocity of P at $t = 12$. [2]

(iii) Given that the total distance covered by P during the T seconds of its motion is 49.5 m, find the value of T. [3]

(iv) Given also that the acceleration of Q from $t = 2$ to $t = 6$ is 1.75 m s^{-2}, find the value of V and hence find the distance between the two particles when they both come to rest at $t = T$. [3]

Answers: (i) 42 m (ii) 3 ms^{-1} (iii) 15 (iv) 7 ms^{-1} and 101 m

N17/43/Q6

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60 A particle moves in a straight line starting from rest from a point O. The acceleration of the particle at time t s after leaving O is a m s^{-2}, where

$$a = 5.4 - 1.62t.$$

(i) Find the positive value of t at which the velocity of the particle is zero, giving your answer as an exact fraction. [4]

(ii) Find the velocity of the particle at $t = 10$ and sketch the velocity-time graph for the first ten seconds of the motion. [3]

(iii) Find the total distance travelled during the first ten seconds of the motion. [5]

N18/41/Q7

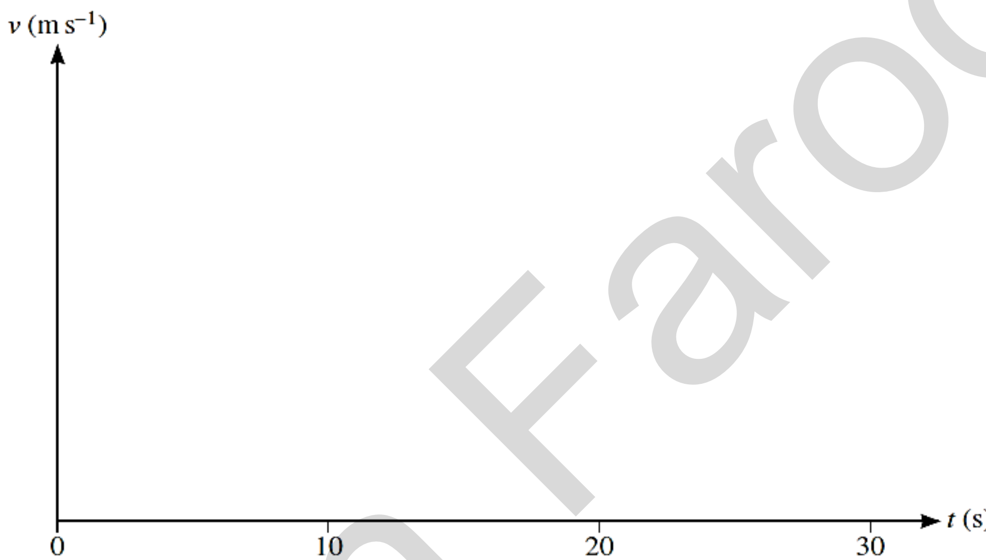
Answers: (i) $t = \frac{20}{3} = 6\frac{2}{3}$

(ii) The v-t graph is an inverted parabola, passing through the points (0,0), ($\frac{20}{3}$, 0) and (10,–27)

(iii) 80 m

61 A runner sets off from a point P at time $t = 0$, where t is in seconds. The runner starts from rest and accelerates at $1.2\,\text{m}\,\text{s}^{-2}$ for 5 s. For the next 12 s the runner moves at constant speed before decelerating uniformly over a period of 3 s, coming to rest at Q. A cyclist sets off from P at time $t = 10$ and accelerates uniformly for 10 s, before immediately decelerating uniformly to rest at Q at time $t = 30$.

(i) Sketch the velocity-time graph for the runner and show that the distance PQ is 96 m. [4]



(ii) Find the magnitude of the acceleration of the cyclist. [3]

Answer: 96 m (AG) *Answer:* $0.96\,\text{ms}^{-2}$ N18/43/Q4

62 A particle moves in a straight line. The particle is initially at rest at a point O on the line. At time t s after leaving O, the acceleration $a\,\text{m}\,\text{s}^{-2}$ of the particle is given by $a = 25 - t^2$ for $0 \leqslant t \leqslant 9$.

(i) Find the maximum velocity of the particle in this time period. [4]

(ii) Find the total distance travelled until the maximum velocity is reached. [2]

13

The acceleration of the particle for $t > 9$ is given by $a = -3t^{-\frac{1}{2}}$.

(iii) Find the velocity of the particle when $t = 25$. [4]

Answer: $83.3\,\text{ms}^{-1}$ *Answer:* 260 m *Answer:* $-30\,\text{ms}^{-1}$ N18/43/Q7

63 A particle P starts from rest at a point O of a straight line and moves along the line. The displacement of the particle at time t s after leaving O is x m, where

$$x = 0.08t^2 - 0.0002t^3.$$

(i) Find the value of t when P returns to O and find the speed of P as it passes through O on its return. [4]

(ii) For the motion of P until the instant it returns to O, find

(a) the total distance travelled, [3]

(b) the average speed. [2]

Answer: (i) 400 s 32 ms^{-1} (ii) (a) 3790 m (ii) (b) 9.48 ms^{-1} N15/43/Q6

64 A particle P moves in a straight line, starting from a point O. The velocity of P, measured in $m\,s^{-1}$, at time t s after leaving O is given by

$$v = 0.6t - 0.03t^2.$$

(i) Verify that, when $t = 5$, the particle is 6.25 m from O. Find the acceleration of the particle at this time. [4]

(ii) Find the values of t at which the particle is travelling at half of its maximum velocity. [6]

Answers: Given: At $t = 5$ the particle is 6.25 m from O Acceleration of the particle when $t = 5$ is 0.3 ms^{-2} N15/41/Q6

Answer: Values of t when the particle is travelling at half of its maximum velocity: $t = 2.93$ s and $t = 17.07$ s

65 A cyclist starts from rest at point A and moves in a straight line with acceleration $0.5\,m\,s^{-2}$ for a distance of 36 m. The cyclist then travels at constant speed for 25 s before slowing down, with constant deceleration, to come to rest at point B. The distance AB is 210 m.

(i) Find the total time that the cyclist takes to travel from A to B. [5]

24 s after the cyclist leaves point A, a car starts from rest from point A, with constant acceleration $4\,m\,s^{-2}$, towards B. It is given that the car overtakes the cyclist while the cyclist is moving with constant speed.

(ii) Find the time that it takes from when the cyclist starts until the car overtakes her. [5]

Answer: Total time taken by the cyclist to travel from A to B is 12 + 25 +8 = 45 seconds N15/41/Q7

Answer: $t^2 - 51t + 594 = 0$ has solutions $t = 18$ (rejected) and $t = 33$. Time taken to overtake is 33 seconds

FORCES AND EQUILIBRIUM

TYPES OF FORCES

Weight

If I hold a brick, I feel it exerting a downward force on my hand. If I let go, the brick will fall to the ground. The force I felt on my hand is now free to pull the brick towards the ground. This force, the gravitational attraction of the earth on the brick is called the **weight** of the brick. It always acts vertically downwards (towards the centre of the earth). The weight of a brick of mass 2 kg is about 20 N (Fig.23.3). This will give some idea of the size of the Newton. In fact, a mass of m kg will have a weight of mg N ($\approx 10m$ N). We shall assume this for the rest of this chapter and discuss it further in the next chapter. Note that weight is the one inescapable force on earth. All bodies have weight.

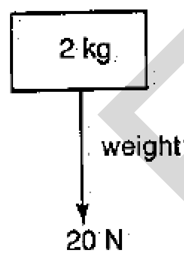


Fig.23.3

Reaction

If I place the brick, of weight 20 N, on a horizontal table, it does not move. We say that the brick is in *equilibrium*. Now it would be unrealistic to assume that the gravitational attraction has ceased, so there must be an opposing force exactly equal to 20 N, acting vertically upwards and also through the CG (Fig.23.4(a)).

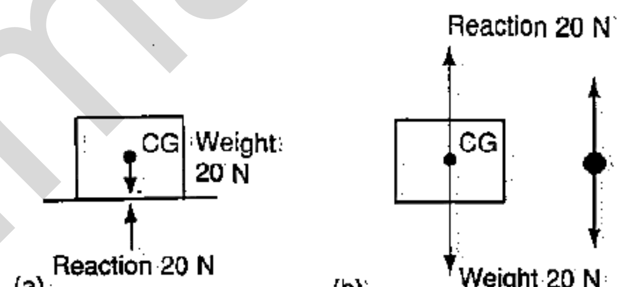


Fig. 23.4

This is the **normal reaction** on the brick from the table, normal in the geometrical sense of being at right angles to the common surface. Sometimes this force is called the normal contact force. In diagrams we show only the forces acting *on* a body, so Fig.23.4(b) shows the forces on the brick. Note also that the dimensions of the body are not relevant in this context. The brick is treated as a **particle**.

Tension

If I suspend the brick by a string (or spring) and the string does not break, the brick is again in equilibrium (Fig.23.5).

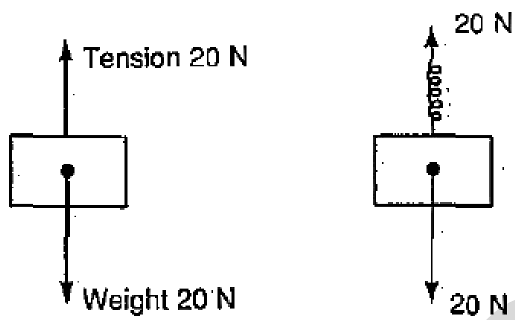


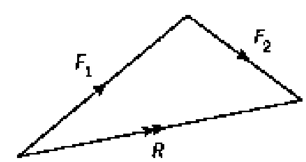
Fig. 23.5

In this case the force which equalizes the weight passes along the string and is called the **tension** in the string. If using a spring, the weight will also stretch the spring until the equilibrium position is reached. (We regard the string as being inextensible, i.e. any stretch is too small to be noted.)

RESULTANTS

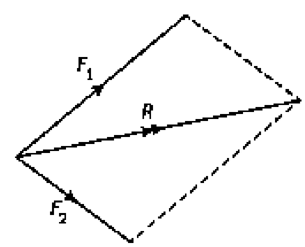
A force has both magnitude and direction, which means it is a vector quantity. The magnitude of a force is measured in newtons (N).

To find the combined effect of a number of forces we can use the triangle rule, placing the start of the second force at the end of the first.



In the diagram, F_1 is combined with F_2 by placing F_2 at the end of F_1. The size and direction of the resultant of the two forces, R, are given by the third side of the triangle.

Or we can use the parallelogram rule, placing both forces with the same starting point and completing a parallelogram.

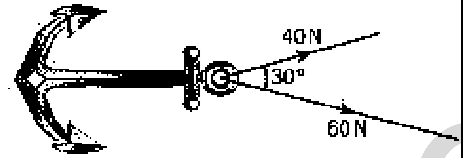


In the diagram the forces F_1 and F_2 are placed together with the same starting point. Their resultant R is the diagonal of the parallelogram.

EXAMPLE 1

An anchor is being pulled using ropes by two sailors with forces of 40 N and 60 N, as shown in the diagram. The angle between the two forces is 30°.

Find the magnitude and direction of the resultant of the two forces.



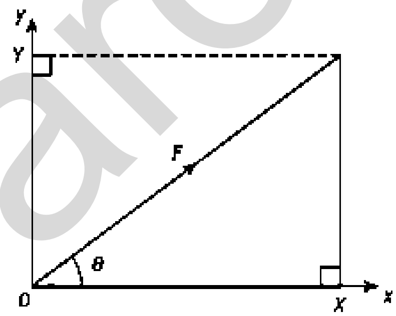
COMPONENTS

Previously we have looked at combining two forces into a single force (called the resultant). We will now look at the reverse process, which involves taking a single force and breaking it up into components. In this process we resolve the force into two components (or resolved parts) in perpendicular directions.

To resolve a force in two perpendicular directions, consider the diagram on the right.

$\cos\theta = \frac{OX}{F}$, giving $OX = F\cos\theta$

$\sin\theta = \frac{OY}{F}$, giving $OY = F\sin\theta$



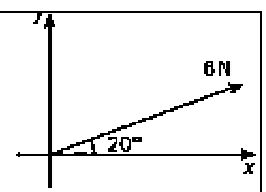
The force in the x-direction, OX, is $F\cos\theta$ and in the y-direction, OY, is $F\sin\theta$.

EXAMPLE 2

Using the diagram, find the components of the given force in the direction of

a) the x-axis

b) the y-axis.

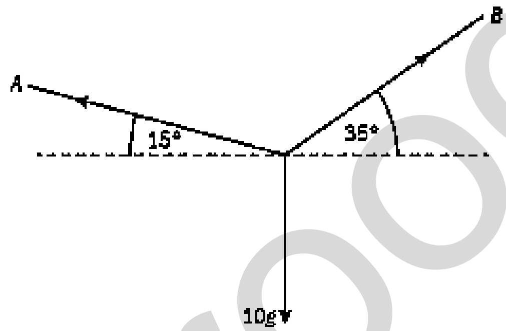


FORCES IN EQUILIBRIUM

<u>EXAMPLE 3</u>

A mass of 10 kg is suspended in equilibrium by two light inextensible strings *A* and *B*, which make angles of 15° and 35°, respectively, to the horizontal, as shown in the diagram. Calculate the tensions in the strings.

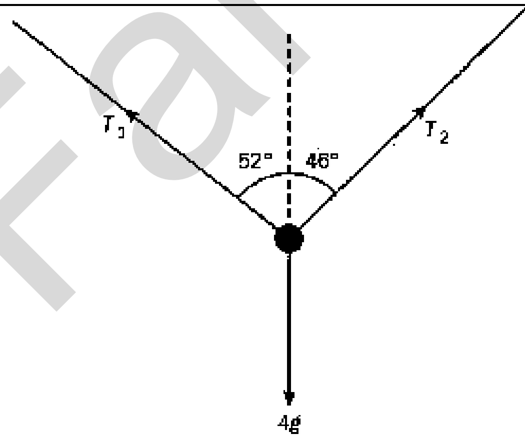
Take $g = 10\,\text{m s}^{-2}$.



<u>EXAMPLE 4</u>

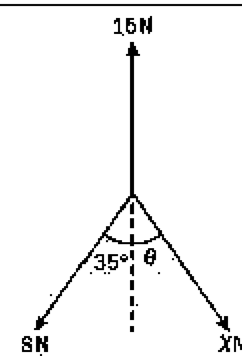
A particle of mass 4 kg is suspended in equilibrium from two light inextensible strings that are at angles of 52° and 46° to the vertical. Calculate the tensions T_1 and T_2 in the two strings.

Take $g = 10\,\text{m s}^{-2}$.



<u>EXAMPLE 5</u>

Three forces of 15 N, 8 N and *X* N on a particle keep it in equilibrium, as shown in the diagram.
Find *X* and θ.



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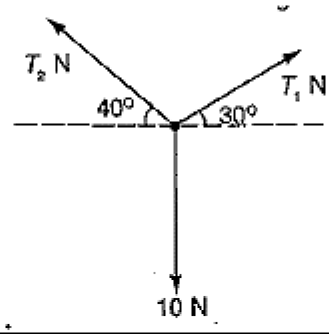
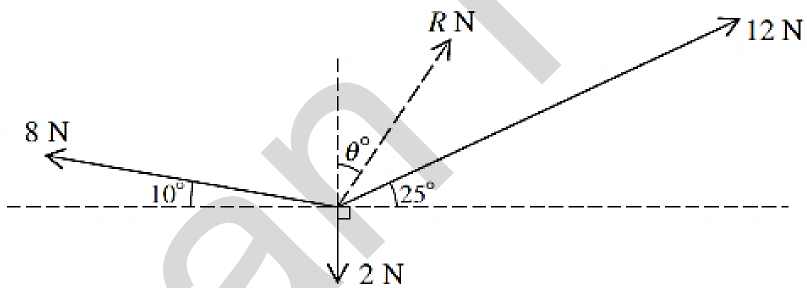
EXAMPLE 6

A particle of weight 10 N is suspended by 2 strings. If these strings make angles of 30°C and 40°C to the horizontal, find the tensions in the string



QUESTION 2

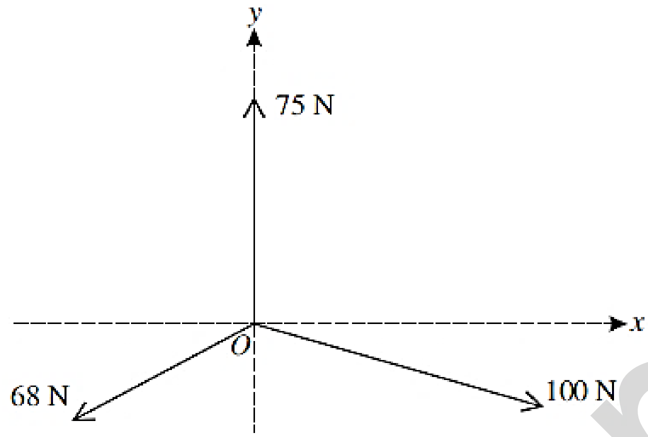
N12/41/Q4



Three coplanar forces of magnitudes 8 N, 12 N and 2 N act at a point. The resultant of the forces has magnitude R N. The directions of the three forces and the resultant are shown in the diagram. Find R and θ. [7]

QUESTION 3

N12/42/Q4



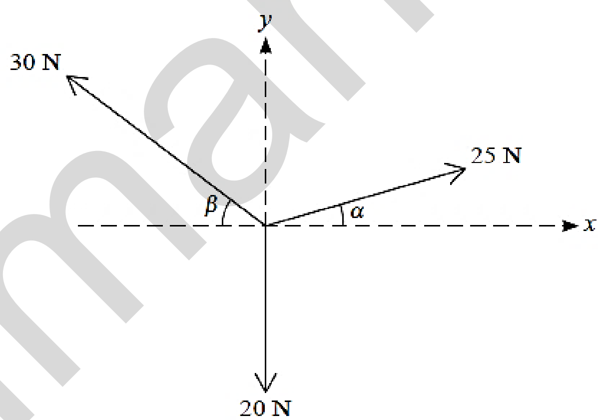
Three coplanar forces of magnitudes 68 N, 75 N and 100 N act at an origin O, as shown in the diagram. The components of the three forces in the positive x-direction are –60 N, 0 N and 96 N, respectively. Find

(i) the components of the three forces in the positive y-direction, [3]

(ii) the magnitude and direction of the resultant of the three forces. [4]

QUESTION 4

N14/42/Q4



Three coplanar forces act at a point. The magnitudes of the forces are 20 N, 25 N and 30 N, and the directions in which the forces act are as shown in the diagram, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$, and $\sin\beta = 0.6$ and $\cos\beta = 0.8$.

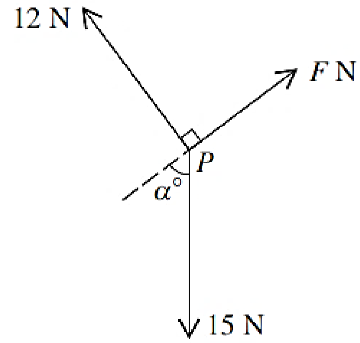
(i) Show that the resultant of the three forces has a zero component in the x-direction. [2]

(ii) Find the magnitude and direction of the resultant of the three forces. [2]

(iii) The force of magnitude 20 N is replaced by another force. The effect is that the resultant force is unchanged in magnitude but reversed in direction. State the magnitude and direction of the replacement force. [1]

QUESTION 5

J12/42/Q4



Three coplanar forces of magnitudes F N, 12 N and 15 N are in equilibrium acting at a point P in the directions shown in the diagram. Find α and F. [4]

FRICTION

I place the brick on a horizontal table and try to push it along (Fig.23.6).

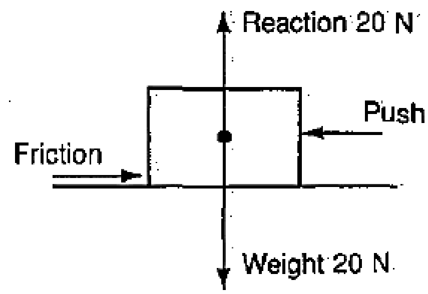


Fig. 23.6

I have to exert a force to do so. There is resistance to my push, called the **frictional resistance** or simply friction. We study this in more detail later. In this situation there are four forces acting on the brick. Note that the weight and normal reaction are still equal and opposite and the brick is in vertical equilibrium. If I push harder, eventually I overcome the frictional force and the horizontal equilibrium will be broken.

EXAMPLE 7

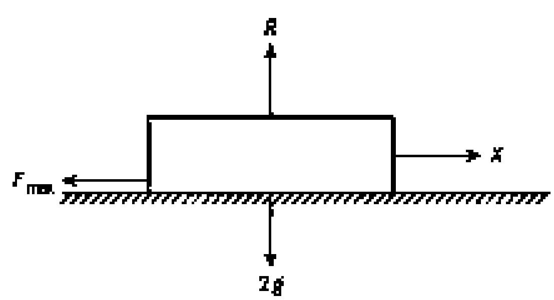
A brick of mass 2 kg is at rest on a rough horizontal surface. The coefficient of friction between the brick and the surface is 0.9.

Calculate the frictional force acting on the brick when a horizontal force *X* is applied to the brick and its magnitude is

a) 10 N b) 15 N c) 35 N.

Calculate the magnitude of any acceleration that takes place due to motion.

Take *g* as 10 ms⁻².



EXAMPLE 8

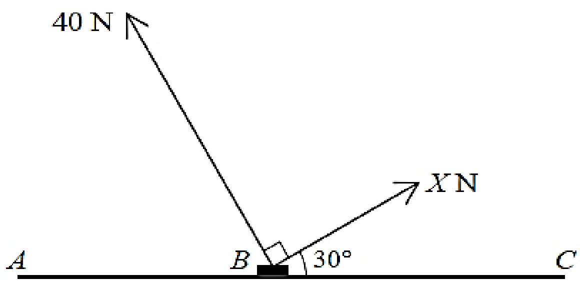
A particle of mass I kg rests on a horizontal floor. The coefficient of friction between the particle and the floor is $\frac{1}{2}$. What force is required just to make the particle move when :

a) Pulling horizontally,

b) Pulling at an angle of 30°C to the horizontal?

EXAMPLE 9

N14/41/Q4



Forces of magnitude X N and 40 N act on a block B of mass 15 kg, which is in equilibrium in contact with a horizontal surface between points A and C on the surface. The forces act in the same vertical plane and in the directions shown in the diagram.

(i) Given that the surface is smooth, find the value of X. [2]

(ii) It is given instead that the surface is rough and that the block is in limiting equilibrium. The frictional force acting on the block has magnitude 10 N in the direction towards A. Find the coefficient of friction between the block and the surface. [5]

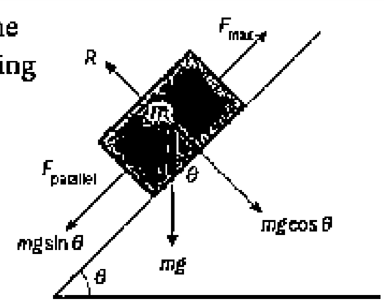
ROUGH INCLINED PLANES

The diagram shows a body of mass m resting on a rough inclined plane that is at an angle of θ to the horizontal. There is no motion, so resolving at right angles to the plane, $R = mg \cos \theta$.

As the parallel force $mg \sin \theta$ is acting down the plane, the frictional force F opposes the direction of this force.

In the system's limiting equilibrium state:

$$F_{max} = \mu R$$
$$= \mu mg \cos \theta$$



EXAMPLE 10

A sledge of mass 10 kg is about to slip as it rests in limiting equilibrium on a rough inclined plane, at 30° to the horizontal. Find the coefficient of friction between the sledge and the plane.

Take g as 10 m s^{-2}. $F_{parallel}$

EXAMPLE 11

A van is carrying fruit from the local market. It breaks down on a steep hill inclined at 30° to the horizontal. Given that the van and fruit have a mass of 1500 kg and the coefficient of friction between the van's wheels and the surface is 0.56, find the force X parallel to the surface that must be applied to the van to prevent motion down the hill.

Take g as $10\,\text{m s}^{-2}$.

EXAMPLE 12

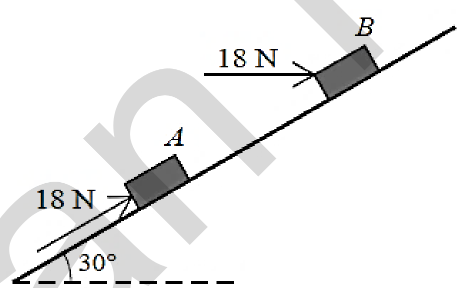
A particle of mass 1 kg is placed on a rough plane inclined at an angle 30° to the horizontal. The coefficient of friction is $\frac{2}{5}$. Find the least force parallel to the plane that is required

(a) to hold the particle at rest,

(b) to make the particle slide up the plane.

QUESTION 13

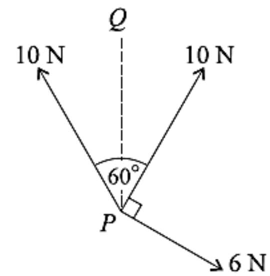
N14/41/Q2



Small blocks A and B are held at rest on a smooth plane inclined at 30° to the horizontal. Each is held in equilibrium by a force of magnitude 18 N. The force on A acts upwards parallel to a line of greatest slope of the plane, and the force on B acts horizontally in the vertical plane containing a line of greatest slope (see diagram). Find the weight of A and the weight of B. [4]

HOMEWORK: FORCES & EQUILIBRIUM VARIANT 42

1



Three coplanar forces of magnitudes 10 N, 10 N and 6 N act at a point P in the directions shown in the diagram. PQ is the bisector of the angle between the two forces of magnitude 10 N.

(i) Find the component of the resultant of the three forces

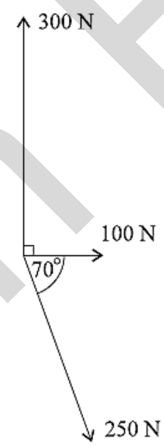
(a) in the direction of PQ, [2]

(b) in the direction perpendicular to PQ. [1]

(ii) Find the magnitude of the resultant of the three forces. [2]

Answers: (i)(a) 14.3 N, (b) 5.20 N; (ii) 15.2 N. J03/Q2

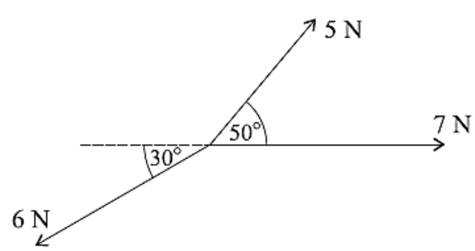
2



Coplanar forces of magnitudes 250 N, 100 N and 300 N act at a point in the directions shown in the diagram. The resultant of the three forces has magnitude R N, and acts at an angle $\alpha°$ anticlockwise from the force of magnitude 100 N. Find R and α. [6]

Answer: R = 197, α = 19.3. J04/Q2

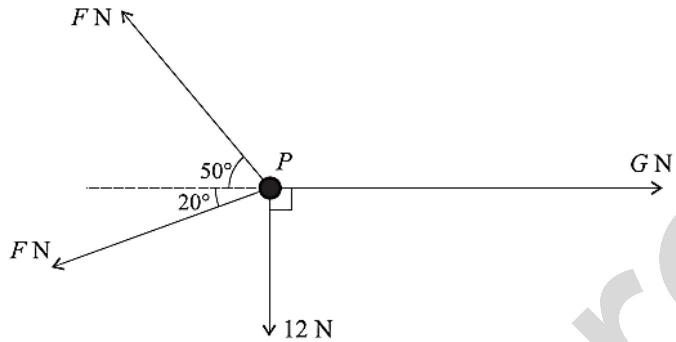
3



Three coplanar forces act at a point. The magnitudes of the forces are 5 N, 6 N and 7 N, and the directions in which the forces act are shown in the diagram. Find the magnitude and direction of the resultant of the three forces. [6]

Answers: 5.09 N, 9.4° anticlockwise from the force of magnitude 7 N. **J05/Q2**

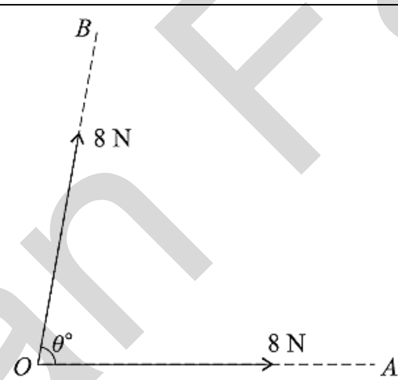
4



A particle P is in equilibrium on a smooth horizontal table under the action of horizontal forces of magnitudes F N, F N, G N and 12 N acting in the directions shown. Find the values of F and G. [6]

Answers: 28.3, 44.8. **J06/Q3**

5



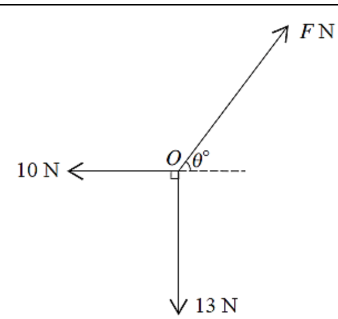
Two forces, each of magnitude 8 N, act at a point in the directions OA and OB. The angle between the forces is $\theta°$ (see diagram). The resultant of the two forces has component 9 N in the direction OA. Find

(i) the value of θ, [2]

(ii) the magnitude of the resultant of the two forces. [3]

Answers: (i) 82.8; (ii) 12 N. **J07/Q2**

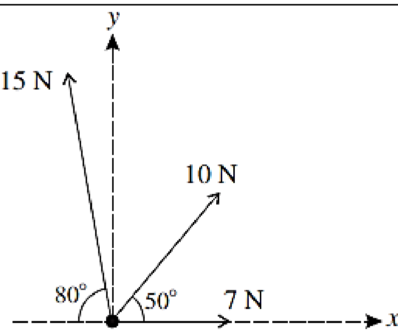
6



Three horizontal forces of magnitudes F N, 13 N and 10 N act at a fixed point O and are in equilibrium. The directions of the forces are as shown in the diagram. Find, in either order, the value of θ and the value of F. [5]

Answers: 52.4, 16.4. **J08/Q3**

7



Forces of magnitudes 7 N, 10 N and 15 N act on a particle in the directions shown in the diagram.

(i) Find the component of the resultant of the three forces

(a) in the x-direction,

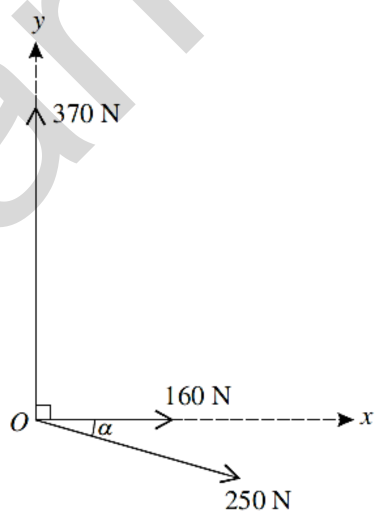
(b) in the y-direction.

[3]

(ii) Hence find the direction of the resultant. [2]

Answers: (i)(a) 10.8 N, (b) 22.4 N; (ii) 64.2° anticlockwise from the x-axis. **J09/Q3**

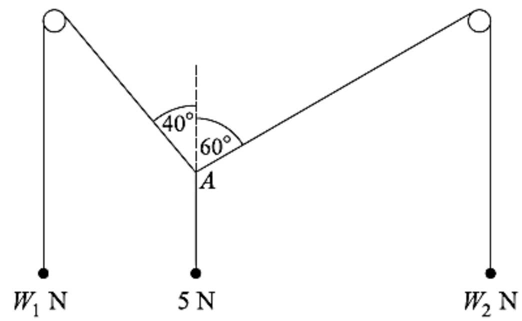
8



Coplanar forces of magnitudes 250 N, 160 N and 370 N act at a point O in the directions shown in the diagram, where the angle α is such that $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$. Calculate the magnitude of the resultant of the three forces. Calculate also the angle that the resultant makes with the x-direction. [7]

Answers: 500 N, 36.9°. **J10/42/Q4**

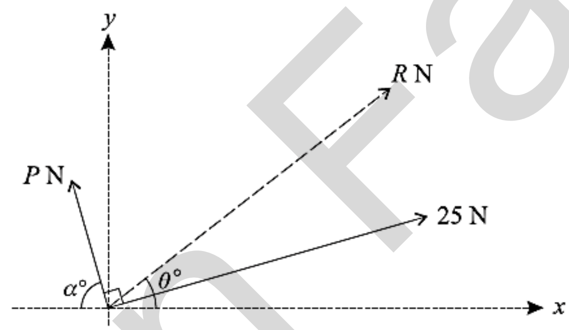
9



Each of three light strings has a particle attached to one of its ends. The other ends of the strings are tied together at a point A. The strings are in equilibrium with two of them passing over fixed smooth horizontal pegs, and with the particles hanging freely. The weights of the particles, and the angles between the sloping parts of the strings and the vertical, are as shown in the diagram. Find the values of W_1 and W_2. [6]

N05/Q3

10



Forces of magnitudes P N and 25 N act at right angles to each other. The resultant of the two forces has magnitude R N and makes an angle of $\theta°$ with the x-axis (see diagram). The force of magnitude P N has components −2.8 N and 9.6 N in the x-direction and the y-direction respectively, and makes an angle of $\alpha°$ with the negative x-axis.

(i) Find the values of P and R. [3]

(ii) Find the value of α, and hence find the components of the force of magnitude 25 N in

(a) the x-direction,

(b) the y-direction.

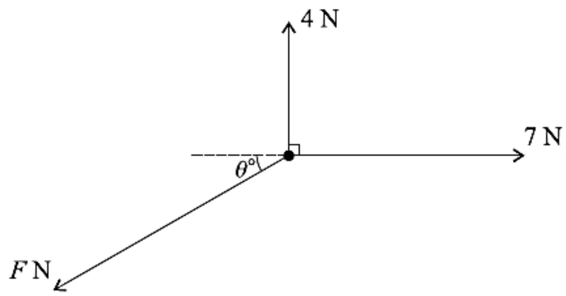
[4]

(iii) Find the value of θ. [3]

Answers: (i) 10, 26.9; (ii)(a) 24 N, (b) 7 N; (iii) 38.1.

N06/Q6

11



A particle is in equilibrium on a smooth horizontal table when acted on by the three horizontal forces shown in the diagram.

(i) Find the values of F and θ. [4]

(ii) The force of magnitude 7 N is now removed. State the magnitude and direction of the resultant of the remaining two forces. [2]

Answers: (i) 8.06, 29.7; (ii) 7 N, direction opposite to that of the force of magnitude 7 N. **N07/Q3**

12

Forces of magnitudes 10 N and 8 N act in directions as shown in the diagram.

(i) Write down in terms of θ the component of the resultant of the two forces

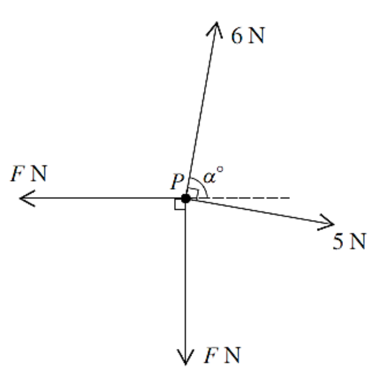
(a) parallel to the force of magnitude 10 N, [1]

(b) perpendicular to the force of magnitude 10 N. [1]

(ii) The resultant of the two forces has magnitude 8 N. Show that $\cos\theta = \frac{5}{8}$. [3]

Answers: (i)(a) $10 - 8\cos\theta$, (b) $8\sin\theta$. **N08/Q1**

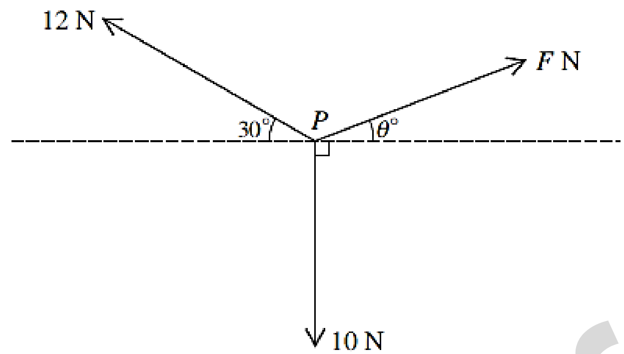
13



A particle P is in equilibrium on a smooth horizontal table under the action of four horizontal forces of magnitudes 6 N, 5 N, F N and F N acting in the directions shown. Find the values of α and F. [6]

N10/42/Q3

14



The three coplanar forces shown in the diagram act at a point P and are in equilibrium.

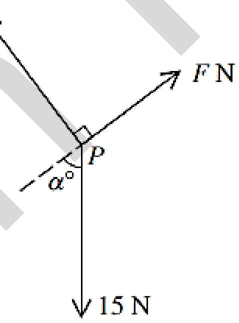
(i) Find the values of F and θ. [6]

(ii) State the magnitude and direction of the resultant force at P when the force of magnitude 12 N is removed. [2]

Answers: **(i)** 11.1, 21.1; **(ii)** 12 N, 30° clockwise from the positive x-axis.

J11/42/Q4

15

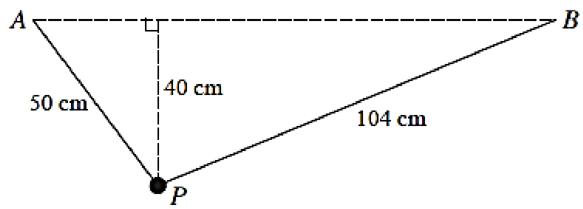


Three coplanar forces of magnitudes F N, 12 N and 15 N are in equilibrium acting at a point P in the directions shown in the diagram. Find α and F. [4]

Answers: α = 53.1°, F = 9 N

J12/42/Q2

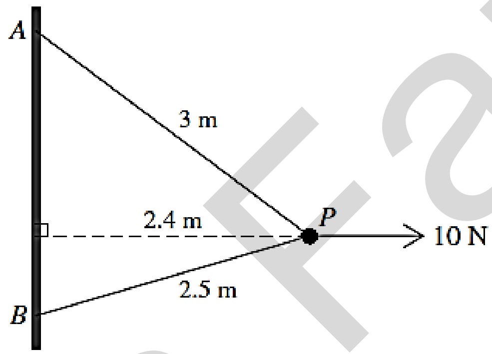
16



A particle P of mass 2.1 kg is attached to one end of each of two light inextensible strings. The other ends of the strings are attached to points A and B which are at the same horizontal level. P hangs in equilibrium at a point 40 cm below the level of A and B, and the strings PA and PB have lengths 50 cm and 104 cm respectively (see diagram). Show that the tension in the string PA is 20 N, and find the tension in the string PB. [5]

Answer: Tension in BP = 13N J13/42/Q3

17

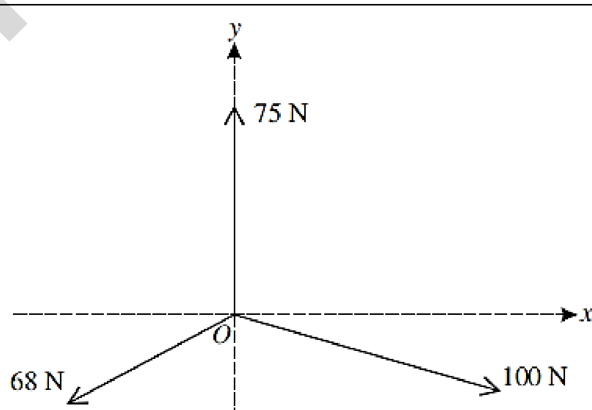


A and B are fixed points of a vertical wall with A vertically above B. A particle P of mass 0.7 kg is attached to A by a light inextensible string of length 3 m. P is also attached to B by a light inextensible string of length 2.5 m. P is maintained in equilibrium at a distance of 2.4 m from the wall by a horizontal force of magnitude 10 N acting on P (see diagram). Both strings are taut, and the 10 N force acts in the plane APB which is perpendicular to the wall. Find the tensions in the strings. [6]

Answer: Tension in AP is 11.9 N. Tension in BP is 0.5 N J14/42/Q3

J14/42/Q4

18



Three coplanar forces of magnitudes 68 N, 75 N and 100 N act at an origin O, as shown in the diagram.

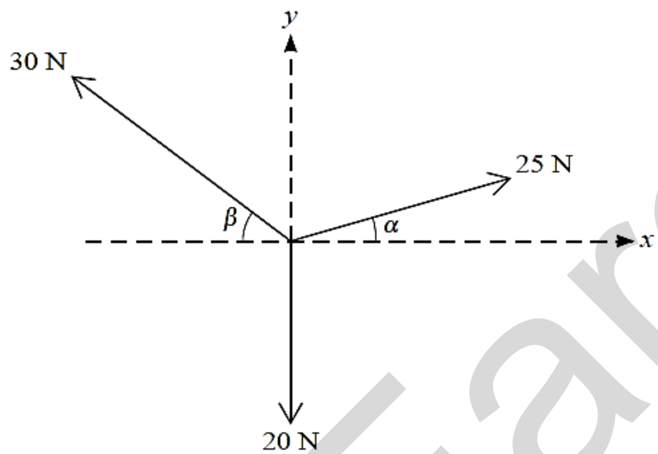
The components of the three forces in the positive x-direction are -60 N, 0 N and 96 N, respectively. Find

(i) the components of the three forces in the positive y-direction, [3]

(ii) the magnitude and direction of the resultant of the three forces. [4]

Answers: (i) -32, 75, -28; (ii) Magnitude 39 N, direction 22.6° anticlockwise from the positive x-axis. **N12/42/Q4**

19



Three coplanar forces act at a point. The magnitudes of the forces are 20 N, 25 N and 30 N, and the directions in which the forces act are as shown in the diagram, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$, and $\sin\beta = 0.6$ and $\cos\beta = 0.8$.

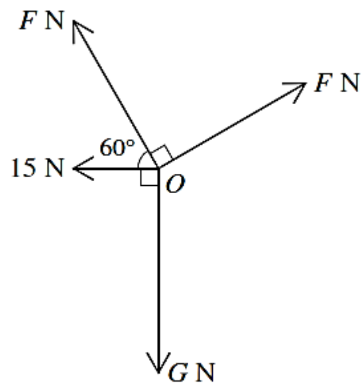
(i) Show that the resultant of the three forces has a zero component in the x-direction. [2]

(ii) Find the magnitude and direction of the resultant of the three forces. [2]

(iii) The force of magnitude 20 N is replaced by another force. The effect is that the resultant force is unchanged in magnitude but reversed in direction. State the magnitude and direction of the replacement force. [1]

Answer: α = 19.4 **N13/42/Q2**

20



Four horizontal forces act at a point O and are in equilibrium. The magnitudes of the forces are F N, G N, 15 N and F N, and the forces act in directions as shown in the diagram.

(i) Show that $F = 41.0$, correct to 3 significant figures. [3]

(ii) Find the value of G. [2]

Answer: Given: $F = 41.0$ *Answer*: $G = 56.0$ or $G = 15(2+\sqrt{3})$ N15/42/Q1

HOMEWORK: FORCES & EQUILIBRIUM VARIANT 41 & 43

1

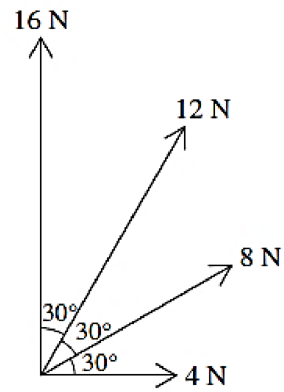


Fig. 1

Four coplanar forces of magnitudes 4 N, 8 N, 12 N and 16 N act at a point. The directions in which the forces act are shown in Fig. 1.

(i) Find the magnitude and direction of the resultant of the four forces. [5]

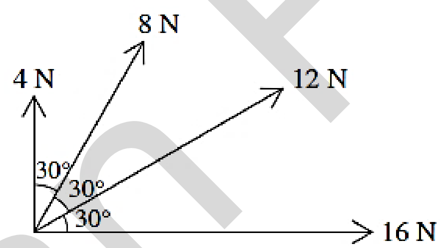


Fig. 2

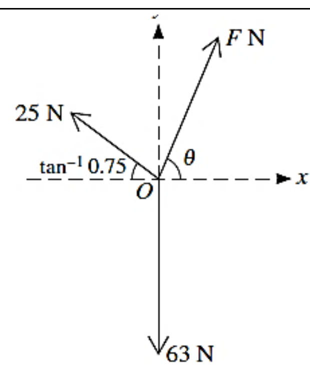
The forces of magnitudes 4 N and 16 N exchange their directions and the forces of magnitudes 8 N and 12 N also exchange their directions (see Fig. 2).

(ii) State the magnitude and direction of the resultant of the four forces in Fig. 2. [2]

Answer: (i) Magnitude of the resultant force = 34.8 N Direction is 60.9° with 4 N force
(ii) Magnitude of the resultant force = 34.8 N Direction is 29.1° with 16 N force

43/J15/5

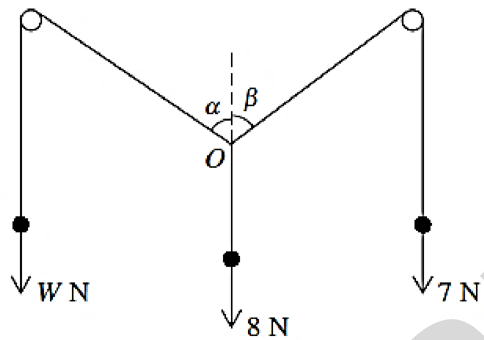
2



Three horizontal forces of magnitudes F N, 63 N and 25 N act at O, the origin of the x-axis and y-axis. The forces are in equilibrium. The force of magnitude F N makes an angle θ anticlockwise with the positive x-axis. The force of magnitude 63 N acts along the negative y-axis. The force of magnitude 25 N acts at $\tan^{-1} 0.75$ clockwise from the negative x-axis (see diagram). Find the value of F and the value of $\tan\theta$. [5]

Answer: F = 52 N or tan θ = 2.4 41/J15/2

3



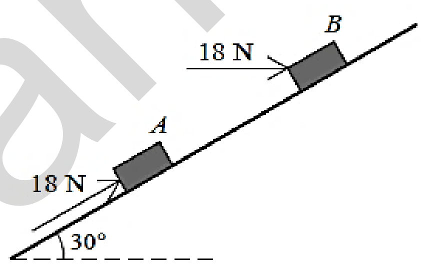
Each of three light inextensible strings has a particle attached to one of its ends. The other ends of the strings are tied together at a point O. Two of the strings pass over fixed smooth pegs and the particles hang freely in equilibrium. The weights of the particles and the angles between the sloping parts of the strings and the vertical are as shown in the diagram. It is given that $\sin\beta = 0.8$ and $\cos\beta = 0.6$.

(i) Show that $W\cos\alpha = 3.8$ and find the value of $W\sin\alpha$. [3]

(ii) Hence find the values of W and α. [3]

Answer: 5.6, 6.77, 55.8 43/N14/3

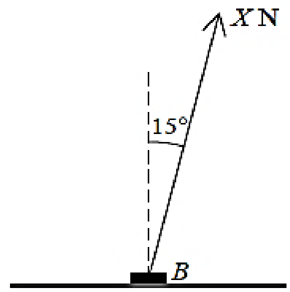
4



Small blocks A and B are held at rest on a smooth plane inclined at 30° to the horizontal. Each is held in equilibrium by a force of magnitude 18 N. The force on A acts upwards parallel to a line of greatest slope of the plane, and the force on B acts horizontally in the vertical plane containing a line of greatest slope (see diagram). Find the weight of A and the weight of B. [4]

Answer: 36, 31.2 41/N14/2

5



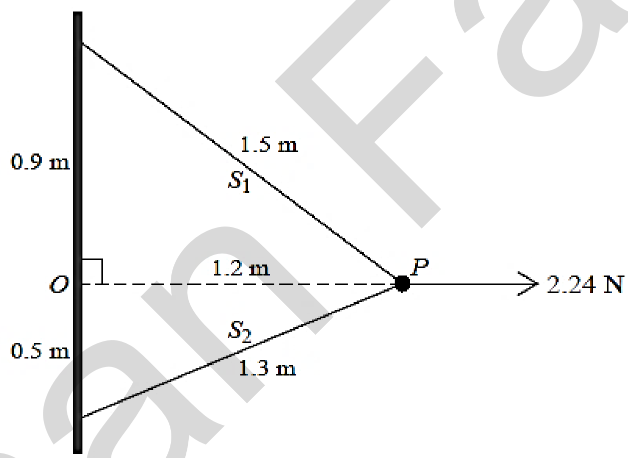
A block B of mass 7 kg is at rest on rough horizontal ground. A force of magnitude X N acts on B at an angle of 15° to the upward vertical (see diagram).

(i) Given that B is in equilibrium find, in terms of X, the normal component of the force exerted on B by the ground. [2]

(ii) The coefficient of friction between B and the ground is 0.4. Find the value of X for which B is in limiting equilibrium. [3]

Answer: 70 – Xcos15°, 43.4 — 43/J14/1

6



A particle P of weight 1.4 N is attached to one end of a light inextensible string S_1 of length 1.5 m, and to one end of another light inextensible string S_2 of length 1.3 m. The other end of S_1 is attached to a wall at the point 0.9 m vertically above a point O of the wall. The other end of S_2 is attached to the wall at the point 0.5 m vertically below O. The particle is held in equilibrium, at the same horizontal level as O, by a horizontal force of magnitude 2.24 N acting away from the wall and perpendicular to it (see diagram). Find the tensions in the strings. [6]

Answer: 2.5 N 0.26 N — 43/J14/3

7 A rough plane is inclined at an angle of α° to the horizontal. A particle of mass 0.25 kg is in equilibrium on the plane. The normal reaction force acting on the particle has magnitude 2.4 N. Find

(i) the value of α, [2]

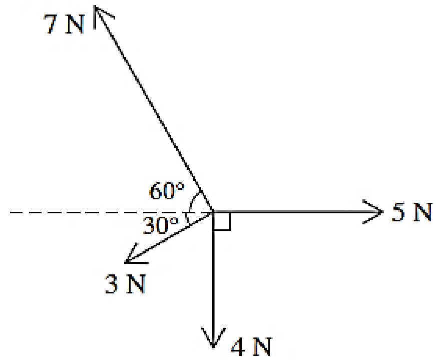
(ii) the least possible value of the coefficient of friction. [2]

41/J14/2

Answer: α = 16.3

Answer: $\mu = \frac{7}{24}$ or 0.292

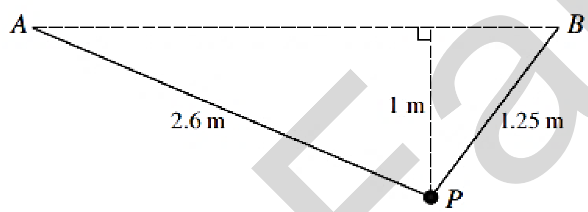
8



Four coplanar forces act at a point. The magnitudes of the forces are 5 N, 4 N, 3 N and 7 N, and the directions in which the forces act are shown in the diagram. Find the magnitude and direction of the resultant of the four forces. [6]

Answer: Resultant = 1.23 N, Direction = 152.9° anticlockwise from the positive x-axis (or equivalent) 41/J14/3

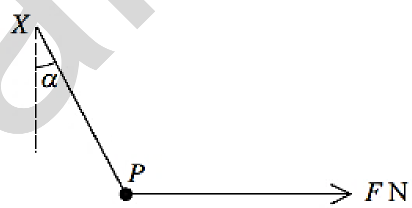
9



A particle P of mass 1.05 kg is attached to one end of each of two light inextensible strings, of lengths 2.6 m and 1.25 m. The other ends of the strings are attached to fixed points A and B, which are at the same horizontal level. P hangs in equilibrium at a point 1 m below the level of A and B (see diagram). Find the tensions in the strings. [6]

Answer: 6.5 N 10 N 43/N13/3

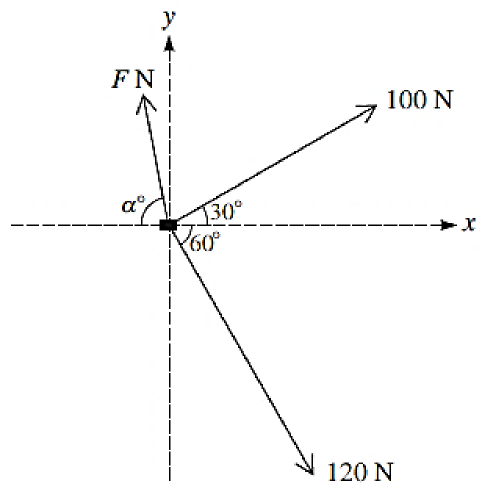
10



A particle P of mass 0.3 kg is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point X. A horizontal force of magnitude F N is applied to the particle, which is in equilibrium when the string is at an angle α to the vertical, where $\tan \alpha = \frac{8}{15}$ (see diagram). Find the tension in the string and the value of F. [4]

Answer: $F = 1.6$ 41/N13/1

11

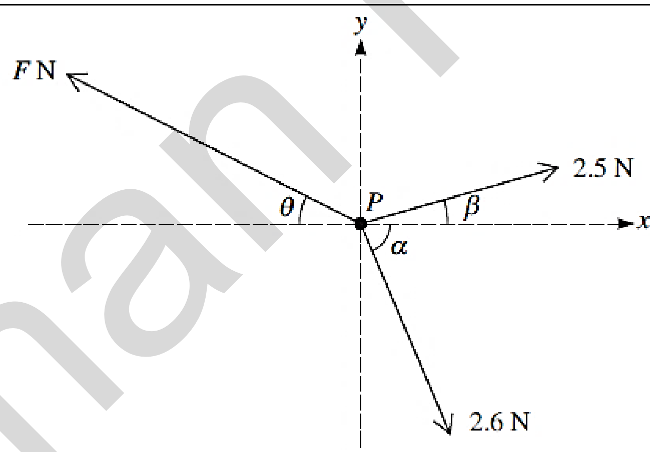


A small box of mass 40 kg is moved along a rough horizontal floor by three men. Two of the men apply horizontal forces of magnitudes 100 N and 120 N, making angles of 30° and 60° respectively with the positive x-direction. The third man applies a horizontal force of magnitude F N making an angle of $\alpha°$ with the negative x-direction (see diagram). The resultant of the three horizontal forces acting on the box is in the positive x-direction and has magnitude 136 N.

(i) Find the values of F and α. [6]

Answer: F=55.0; α=78.9; 43/J13/6

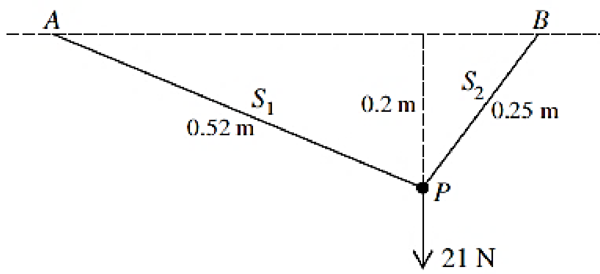
12



A particle P of mass 0.5 kg lies on a smooth horizontal plane. Horizontal forces of magnitudes F N, 2.5 N and 2.6 N act on P. The directions of the forces are as shown in the diagram, where $\tan\alpha = \frac{12}{5}$ and $\tan\beta = \frac{7}{24}$.

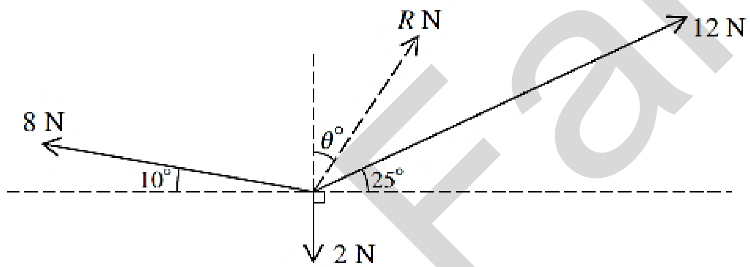
(i) Given that P is in equilibrium, find the values of F and $\tan\theta$. [6]

Answer: 3.80 and 0.5 41/J13/6

13 

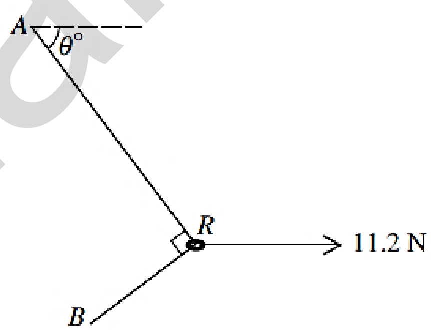
A particle P of weight 21 N is attached to one end of each of two light inextensible strings, S_1 and S_2, of lengths 0.52 m and 0.25 m respectively. The other end of S_1 is attached to a fixed point A, and the other end of S_2 is attached to a fixed point B at the same horizontal level as A. The particle P hangs in equilibrium at a point 0.2 m below the level of AB with both strings taut (see diagram). Find the tension in S_1 and the tension in S_2. [6]

Answer. $S_1 = 13$ N, $S_2 = 20$ N. 43/N12/4

14 

Three coplanar forces of magnitudes 8 N, 12 N and 2 N act at a point. The resultant of the forces has magnitude R N. The directions of the three forces and the resultant are shown in the diagram. Find R and θ. [7]

Answer: $R = 5.37$, $\theta = 33.9°$. 41/N12/4

15 

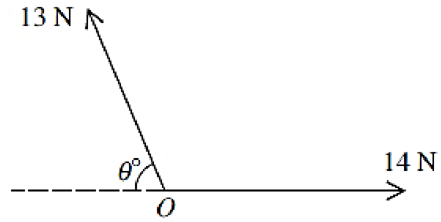
A smooth ring R of mass 0.16 kg is threaded on a light inextensible string. The ends of the string are attached to fixed points A and B. A horizontal force of magnitude 11.2 N acts on R, in the same vertical plane as A and B. The ring is in equilibrium. The string is taut with angle $ARB = 90°$, and the part AR of the string makes an angle of $\theta°$ with the horizontal (see diagram). The tension in the string is T N.

(i) Find two simultaneous equations involving $T \sin\theta$ and $T \cos\theta$. [3]

(ii) Hence find T and θ. [3]

Answer: ii) 8 and 53.1

16



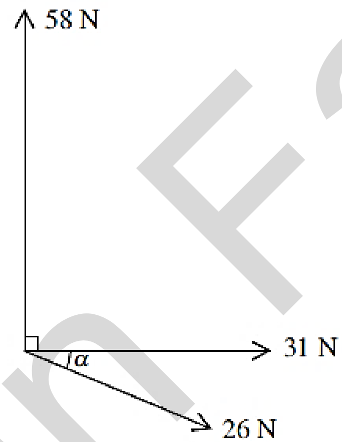
Forces of magnitudes 13 N and 14 N act at a point O in the directions shown in the diagram. The resultant of these forces has magnitude 15 N. Find

(i) the value of θ, **[3]**

(ii) the component of the resultant in the direction of the force of magnitude 14 N. **[2]**

41/J12/2

17

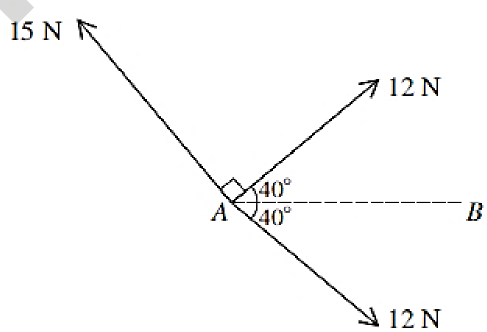


Coplanar forces of magnitudes 58 N, 31 N and 26 N act at a point in the directions shown in the diagram. Given that $\tan\alpha = \frac{5}{12}$, find the magnitude and direction of the resultant of the three forces. **[6]**

Answers: 73 N, 41.1° to **i** direction

43/N11/2

18



Three coplanar forces of magnitudes 15 N, 12 N and 12 N act at a point A in directions as shown in the diagram.

(i) Find the component of the resultant of the three forces

(a) in the direction of AB,

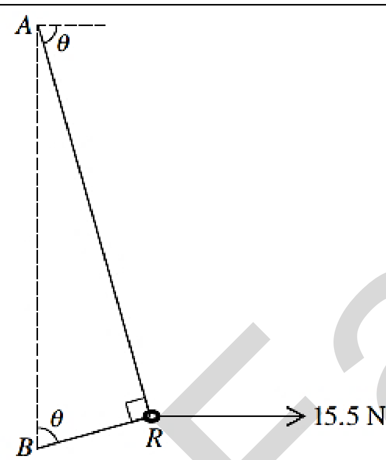
(b) perpendicular to AB.

[3]

(ii) Hence find the magnitude and direction of the resultant of the three forces. [3]

Answers: (i) (a) 8.74 N; (b) 11.5 N; (ii) 14.4 N, 52.7° to the **i** direction 41/N11/3

19



A small smooth ring R of weight 8.5 N is threaded on a light inextensible string. The ends of the string are attached to fixed points A and B, with A vertically above B. A horizontal force of magnitude 15.5 N acts on R so that the ring is in equilibrium with angle $ARB = 90°$. The part AR of the string makes an angle θ with the horizontal and the part BR makes an angle θ with the vertical (see diagram). The tension in the string is T N. Show that $T \sin\theta = 12$ and $T \cos\theta = 3.5$ and hence find θ. [6]

Answer: 73.7°. 41/J11/3

---

20 A force of magnitude F N acts in a horizontal plane and has components 27.5 N and −24 N in the x-direction and the y-direction respectively. The force acts at an angle of $\alpha°$ below the x-axis.

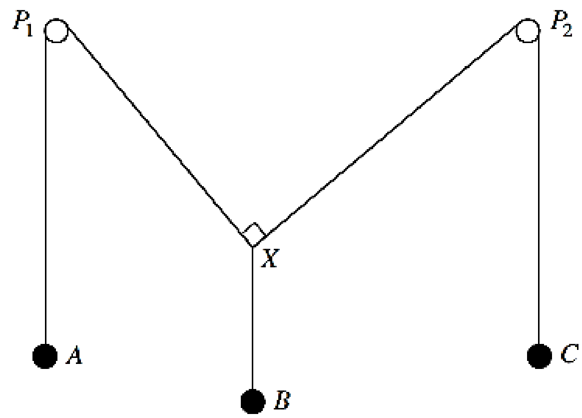
(i) Find the values of F and α. [4]

A second force, of magnitude 87.6 N, acts in the same plane at 90° anticlockwise from the force of magnitude F N. The resultant of the two forces has magnitude R N and makes an angle of $\theta°$ with the positive x-axis.

(ii) Find the values of R and θ. [3]

Answers: (i) 36.5, 41.1; (ii) 94.9, 26.3. 43/N10/5

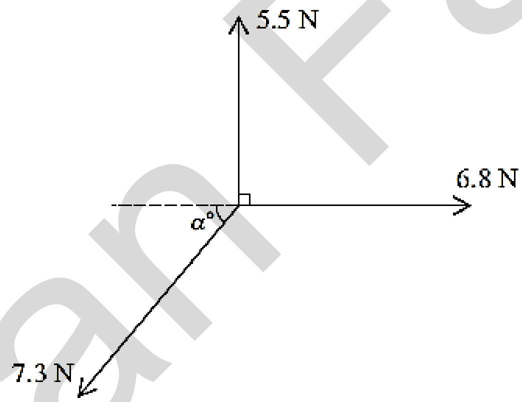
21



The diagram shows three particles A, B and C hanging freely in equilibrium, each being attached to the end of a string. The other ends of the three strings are tied together and are at the point X. The strings carrying A and C pass over smooth fixed horizontal pegs P_1 and P_2 respectively. The weights of A, B and C are 5.5 N, 7.3 N and W N respectively, and the angle P_1XP_2 is a right angle. Find the angle AP_1X and the value of W. [5]

Answer: 4.8 and 41.1 degrees 41/N10/3

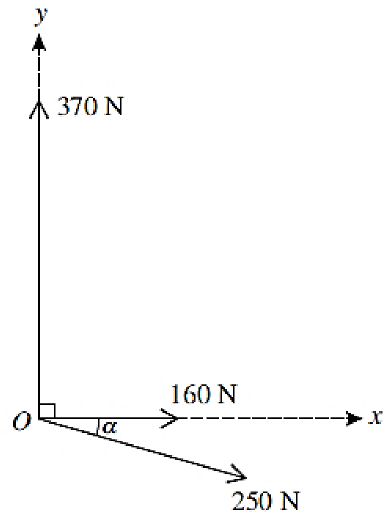
22



Three coplanar forces act at a point. The magnitudes of the forces are 5.5 N, 6.8 N and 7.3 N, and the directions in which the forces act are as shown in the diagram. Given that the resultant of the three forces is in the same direction as the force of magnitude 6.8 N, find the value of α and the magnitude of the resultant. [4]

Answers: 48.9, 2N. 43/J10/1

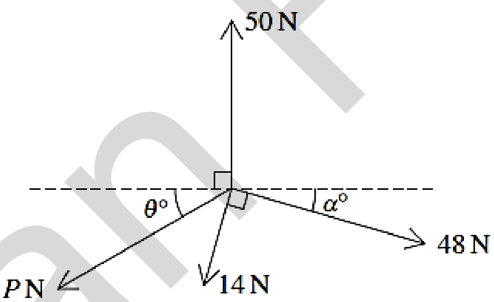
23



Coplanar forces of magnitudes 250 N, 160 N and 370 N act at a point O in the directions shown in the diagram, where the angle α is such that $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$. Calculate the magnitude of the resultant of the three forces. Calculate also the angle that the resultant makes with the x-direction. [7]

Answer: 500N, 36.9 41/J10/4

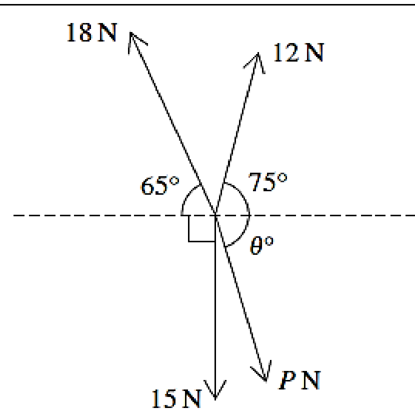
24



Coplanar forces of magnitudes 50 N, 48 N, 14 N and P N act at a point in the directions shown in the diagram. The system is in equilibrium. Given that $\tan\alpha = \frac{7}{24}$, find the values of P and θ. [6]

Answers: $P = 48.1$ $\theta = 28.7$ J16/41/Q4

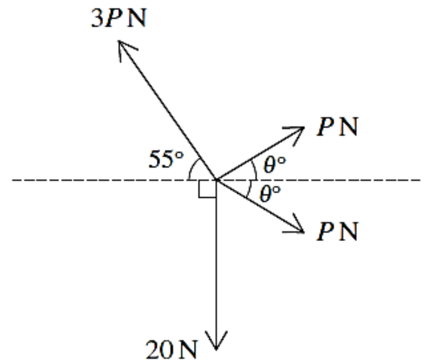
25



The coplanar forces shown in the diagram are in equilibrium. Find the values of P and θ. [6]

Answer: 13.7 70.8 J16/43/Q3

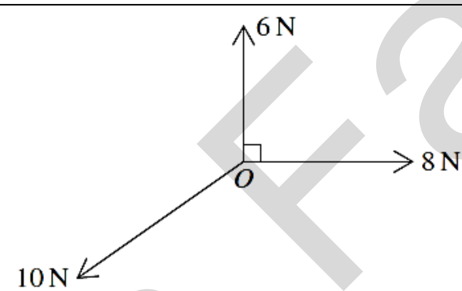
26



The four coplanar forces shown in the diagram are in equilibrium. Find the values of P and θ. [5]

Answer: 8.14 N 30.6° J17/43/Q2

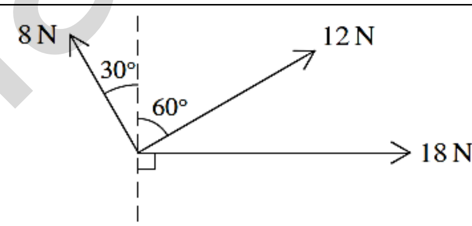
27



The diagram shows three coplanar forces acting at the point O. The magnitudes of the forces are 6 N, 8 N and 10 N. The angle between the 6 N force and the 8 N force is 90°. The forces are in equilibrium. Find the other angles between the forces. [4]

Answers: The angle between the 6 N force and 10 N force is 126.9° (to 1 decimal places)
The angle between the 8 N force and 10 N force is 143.1° (to 1 decimal places) J18/41/Q2

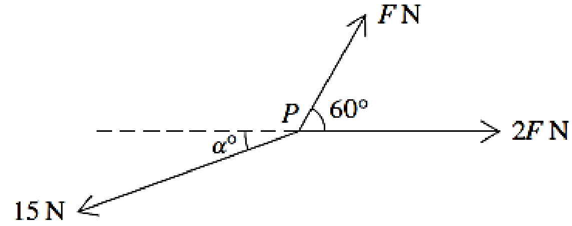
28



Coplanar forces of magnitudes 8 N, 12 N and 18 N act at a point in the directions shown in the diagram. Find the magnitude and direction of the single additional force acting at the same point which will produce equilibrium. [6]

Answer: 27.6 N, 27.9° below 'negative *x*-axis' J18/43/Q3

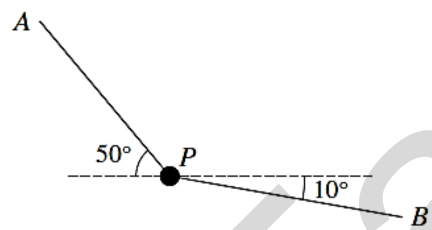
29



Three coplanar forces of magnitudes F N, $2F$ N and 15 N act at a point P, as shown in the diagram. Given that the forces are in equilibrium, find the values of F and α. [6]

Answers: F = 5.67 and α = 19.1 N16/41/Q4

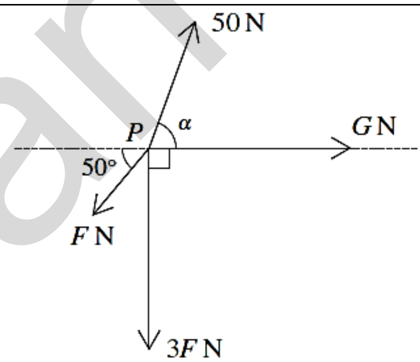
30



The diagram shows a small object P of mass 20 kg held in equilibrium by light ropes attached to fixed points A and B. The rope PA is inclined at an angle of 50° above the horizontal, the rope PB is inclined at an angle of 10° below the horizontal, and both ropes are in the same vertical plane. Find the tension in the rope PA and the tension in the rope PB. [5]

Answers: Tension in rope PA is 306 N Tension in rope PB is 200 N N16/43/Q2

31



Coplanar forces, of magnitudes F N, $3F$ N, G N and 50 N, act at a point P, as shown in the diagram.

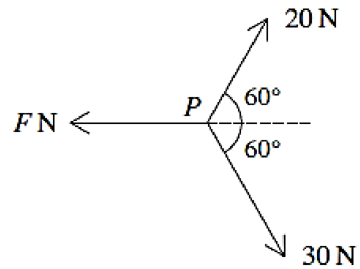
(i) Given that $F = 0$, $G = 75$ and $\alpha = 60°$, find the magnitude and direction of the resultant force. [4]

(ii) Given instead that $G = 0$ and the forces are in equilibrium, find the values of F and α. [5]

Answers: Magnitude of the resultant force is 109 N (to 3sf) N17/41/Q6
The direction of the resultant is 23.4° anticlockwise from the positive x-axis (to 1dp)

Answers: F = 13.1 (to 3sf), α = 80.3 (to 1dp)

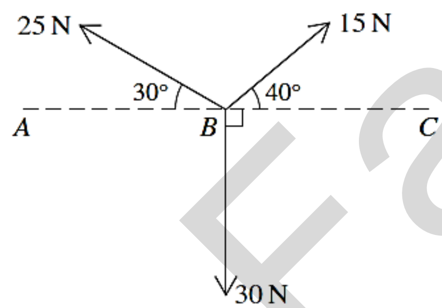
32



Three coplanar forces of magnitudes F N, 20 N and 30 N act at a point P, as shown in the diagram. The resultant of the three forces acts in a direction perpendicular to the force of magnitude F N. Find the value of F. [3]

Answer: 25 N17/43/Q1

33



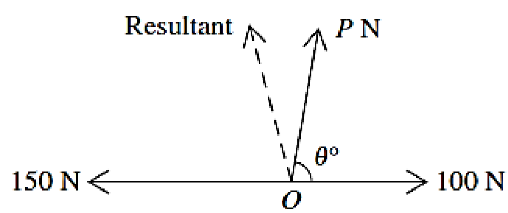
Coplanar forces, of magnitudes 15 N, 25 N and 30 N, act at a point B on the line ABC in the directions shown in the diagram.

(i) Find the magnitude and direction of the resultant force. [6]

(ii) The force of magnitude 15 N is now replaced by a force of magnitude F N acting in the same direction. The new resultant force has zero component in the direction BC. Find the value of F, and find also the magnitude and direction of the new resultant force. [3]

Answers: **(i)** 12.8 N; Direction: angle of 37.7° (to 3 sf) below the negative x-axis
(ii) F = 28.3 (to 3 sf) The magnitude of the new force is 0.667 N (to 3 sf) in a direction vertically upwards N18/41/Q5

34



Three horizontal forces of magnitudes 150 N, 100 N and P N have directions as shown in the diagram. The resultant of the three forces is shown by the broken line in the diagram. This resultant has magnitude 120 N and makes an angle 75° with the 150 N force. Find the values of P and θ. [7]

Answer: 117 80.7 N15/43/Q3

NEWTON'S LAWS OF MOTION

Newton's first law states that every body remains in a state of rest or of uniform motion in a straight line unless an external force acts on it.

Newton's second law states that the resultant of the forces acting on a body is equal to the mass of the body multiplied by its acceleration in the direction of that force.

Newton's third law states that to every action there is an equal and opposite reaction.

Note: In mechanics we talk about **bodies**, for example, a cricket ball, a car or a spacecraft. Strictly, Newton's three laws apply to **particles**; however, we use them for most cases involving bodies because they work well enough in most situations and are easy to calculate.

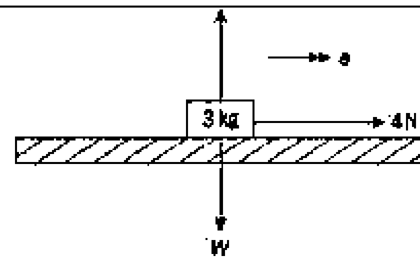
<u>EXAMPLE 1</u>

A particle of mass 3 kg rests on a smooth plane. It is pulled by a horizontal force of 4 N.

Taking the value of g as 10 m s^{-2}, calculate

a) the horizontal acceleration of the particle, a

b) the normal reaction, R.



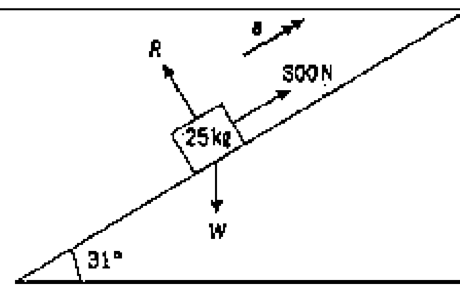
<u>EXAMPLE 2</u>

A particle of mass 25 kg rests on a smooth slope that is at 31° to the horizontal. It is pulled by a force of 300 N up the plane.

Taking the value of g as 10 m s^{-2}, calculate

a) the acceleration of the particle up the plane, a.

b) the normal reaction, R.



EXAMPLE 3

J13/42/Q1

A string is attached to a block of weight 30 N, which is in contact with a rough horizontal plane. When the string is horizontal and the tension in it is 24 N, the block is in limiting equilibrium.

(i) Find the coefficient of friction between the block and the plane. [2]

The block is now in motion and the string is at an angle of 30° upwards from the plane. The tension in the string is 25 N.

(ii) Find the acceleration of the block. [4]

EXAMPLE 4

N13/41/Q4

Particles P and Q are moving in a straight line on a rough horizontal plane. The frictional forces are the only horizontal forces acting on the particles.

(i) Find the deceleration of each of the particles given that the coefficient of friction between P and the plane is 0.2, and between Q and the plane is 0.25. [2]

At a certain instant, P passes through the point A and Q passes through the point B. The distance AB is 5 m. The velocities of P and Q at A and B are 8 m s^{-1} and 3 m s^{-1}, respectively, both in the direction AB.

(ii) Find the speeds of P and Q immediately before they collide. [5]

EXAMPLE 5

J12/42/Q6

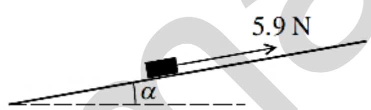


Fig. 1

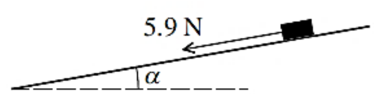


Fig. 2

A block of weight 6.1 N is at rest on a plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{11}{60}$. The coefficient of friction between the block and the plane is μ. A force of magnitude 5.9 N acting parallel to a line of greatest slope is applied to the block.

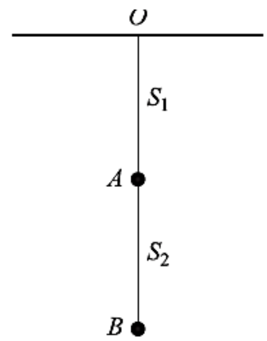
(i) When the force acts up the plane (see Fig. 1) the block remains at rest. Show that $\mu \geqslant \frac{4}{5}$. [5]

(ii) When the force acts down the plane (see Fig. 2) the block slides downwards. Show that $\mu < \frac{7}{6}$. [2]

(iii) Given that the acceleration of the block is 1.7 m s^{-2} when the force acts down the plane, find the value of μ. [2]

HOMEWORK: LAWS OF MOTION VARIANT 42

1



S_1 and S_2 are light inextensible strings, and A and B are particles each of mass 0.2 kg. Particle A is suspended from a fixed point O by the string S_1, and particle B is suspended from A by the string S_2. The particles hang in equilibrium as shown in the diagram.

(i) Find the tensions in S_1 and S_2. [3]

The string S_1 is cut and the particles fall. The air resistance acting on A is 0.4 N and the air resistance acting on B is 0.2 N.

(ii) Find the acceleration of the particles and the tension in S_2. [5]

Answers: (i) 4 N in S_1, 2N in S_2; (ii) 8.5 ms^{-2}, 0.1 N. **J03/Q5**

2 A small block of mass 0.15 kg moves on a horizontal surface. The coefficient of friction between the block and the surface is 0.025.

(i) Find the frictional force acting on the block. [2]

(ii) Show that the deceleration of the block is 0.25 m s^{-2}. [2]

The block is struck from a point A on the surface and, 4 s later, it hits a boundary board at a point B. The initial speed of the block is 5.5 m s^{-1}.

(iii) Find the distance AB. [2]

The block rebounds from the board with a speed of 3.5 m s^{-1} and moves along the line BA. Find

(iv) the speed with which the block passes through A, [2]

(v) the total distance moved by the block, from the instant when it was struck at A until the instant when it comes to rest. [2]

Answers: (i) 0.0375 N; (iii) 20 m; (iv) 1.5 ms^{-1}; (v) 44.5 m. **J03/Q6**

3 A and B are points on the same line of greatest slope of a rough plane inclined at 30° to the horizontal. A is higher up the plane than B and the distance AB is 2.25 m. A particle P, of mass m kg, is released from rest at A and reaches B 1.5 s later. Find the coefficient of friction between P and the plane. [6]

Answer: 0.346. **J05/Q3**

4 A car of mass 1200 kg travels on a horizontal straight road with constant acceleration $a\,\mathrm{m\,s^{-2}}$.

(i) Given that the car's speed increases from $10\,\mathrm{m\,s^{-1}}$ to $25\,\mathrm{m\,s^{-1}}$ while travelling a distance of 525 m, find the value of a. [2]

The car's engine exerts a constant driving force of 900 N. The resistance to motion of the car is constant and equal to R N.

(ii) Find R. [2]

- *Answers*: (i) 0.5; (ii) 300. J06/Q1

5 **(i)** A particle P of mass 1.2 kg is released from rest at the top of a slope and starts to move. The slope has length 4 m and is inclined at 25° to the horizontal. The coefficient of friction between P and the slope is $\frac{1}{4}$. Find

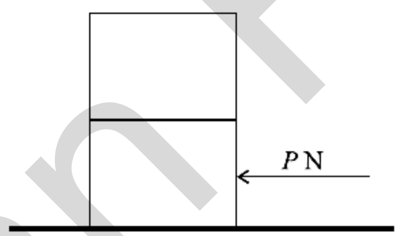
(a) the frictional component of the contact force on P, [2]

(b) the acceleration of P, [2]

(c) the speed with which P reaches the bottom of the slope. [2]

Answers: **(i)(a)** 2.72 N, **(b)** $1.96\mathrm{ms^{-2}}$, **(c)** $3.96\ \mathrm{ms^{-1}}$; **(ii)(a)** 36 J, **(b)** $8.70\ \mathrm{ms^{-1}}$. N02/Q6

6



Two identical boxes, each of mass 400 kg, are at rest, with one on top of the other, on horizontal ground. A horizontal force of magnitude P newtons is applied to the lower box (see diagram). The coefficient of friction between the lower box and the ground is 0.75 and the coefficient of friction between the two boxes is 0.4.

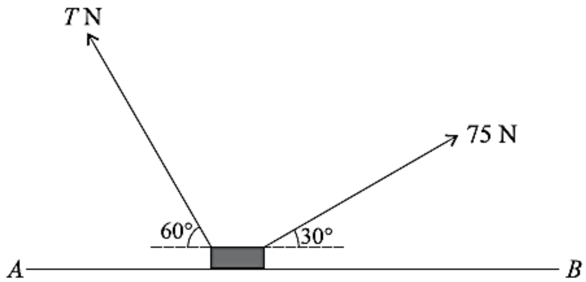
(i) Show that the boxes will remain at rest if $P \leqslant 6000$. [2]

The boxes start to move with acceleration $a\,\mathrm{m\,s^{-2}}$.

(ii) Given that no sliding takes place between the boxes, show that $a \leqslant 4$ and deduce the maximum possible value of P. [7]

Answer: **(ii)** $P_{max} = 9200$ N. N04/Q6

7



Two light strings are attached to a block of mass 20 kg. The block is in equilibrium on a horizontal surface AB with the strings taut. The strings make angles of 60° and 30° with the horizontal, on either side of the block, and the tensions in the strings are T N and 75 N respectively (see diagram).

(i) Given that the surface is smooth, find the value of T and the magnitude of the contact force acting on the block. [5]

(ii) It is given instead that the surface is rough and that the block is on the point of slipping. The frictional force on the block has magnitude 25 N and acts towards A. Find the coefficient of friction between the block and the surface. [6]

Answers: (i) 130, 50 N; (ii) 0.268. **J07/Q7**

8

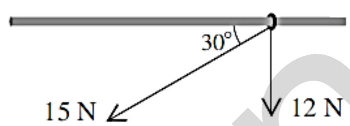


Fig. 1

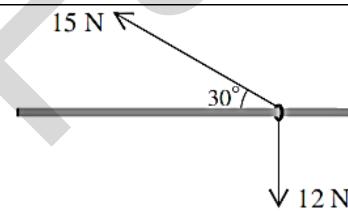


Fig. 2

A small ring of weight 12 N is threaded on a fixed rough horizontal rod. A light string is attached to the ring and the string is pulled with a force of 15 N at an angle of 30° to the horizontal.

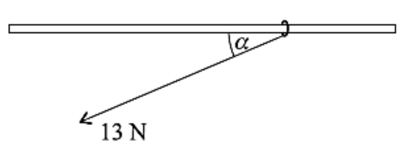
(i) When the angle of 30° is **below** the horizontal (see Fig. 1), the ring is in limiting equilibrium. Show that the coefficient of friction between the ring and the rod is 0.666, correct to 3 significant figures. [5]

(ii) When the angle of 30° is **above** the horizontal (see Fig. 2), the ring is moving with acceleration $a\ \text{m s}^{-2}$. Find the value of a. [4]

N09/42/Q5

Answer: **(ii)** 8.33.

9



A ring of mass 1.1 kg is threaded on a fixed rough horizontal rod. A light string is attached to the ring and the string is pulled with a force of magnitude 13 N at an angle α below the horizontal, where $\tan \alpha = \frac{5}{12}$ (see diagram). The ring is in equilibrium.

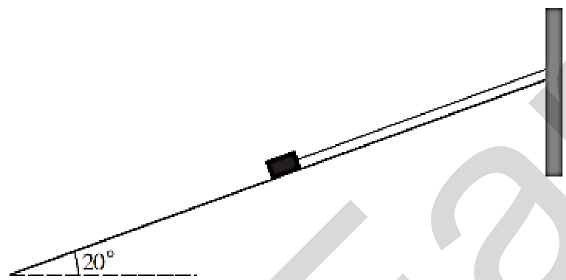
(i) Find the frictional component of the contact force on the ring. [2]

(ii) Find the normal component of the contact force on the ring. [2]

(iii) Given that the equilibrium of the ring is limiting, find the coefficient of friction between the ring and the rod. [1]

Answers: (i) 12 N; (ii) 16 N; (iii) 0.75. **J04/Q1**

10



A block of mass 8 kg is at rest on a plane inclined at 20° to the horizontal. The block is connected to a vertical wall at the top of the plane by a string. The string is taut and parallel to a line of greatest slope of the plane (see diagram).

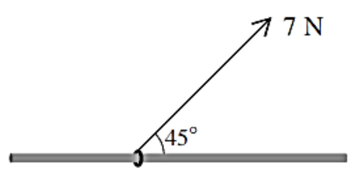
(i) Given that the tension in the string is 13 N, find the frictional and normal components of the force exerted on the block by the plane. [4]

The string is cut; the block remains at rest, but is on the point of slipping down the plane.

(ii) Find the coefficient of friction between the block and the plane. [2]

Answers: (i) 14.4 N, 75.2 N; (ii) 0.364. **J09/Q4**

11



A small ring of mass 0.8 kg is threaded on a rough rod which is fixed horizontally. The ring is in equilibrium, acted on by a force of magnitude 7 N pulling upwards at 45° to the horizontal (see diagram).

(i) Show that the normal component of the contact force acting on the ring has magnitude 3.05 N, correct to 3 significant figures. [2]

(ii) The ring is in limiting equilibrium. Find the coefficient of friction between the ring and the rod. [3]

Answers: **(i)** 20 s; **(ii)** 80 s; **(iii)** 4 ms^{-1}; **(iv)** 1170 m. J10/42/Q3

12

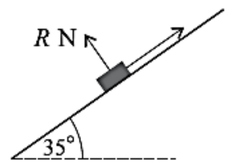


Fig. 1

A force, whose direction is upwards parallel to a line of greatest slope of a plane inclined at 35° to the horizontal, acts on a box of mass 15 kg which is at rest on the plane. The normal component of the contact force on the box has magnitude R newtons (see Fig. 1).

(i) Show that $R = 123$, correct to 3 significant figures. [1]

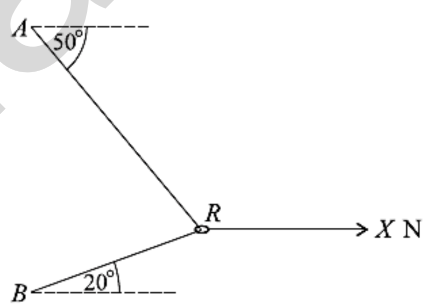
Fig. 2

When the force parallel to the plane acting on the box has magnitude X newtons the box is about to move *down* the plane, and when this force has magnitude $5X$ newtons the box is about to move *up* the plane (see Fig. 2).

(ii) Find the value of X and the coefficient of friction between the box and the plane. [7]

Answer: **(ii)** X = 28.7, coefficient of friction = 0.467. N02/Q5

13



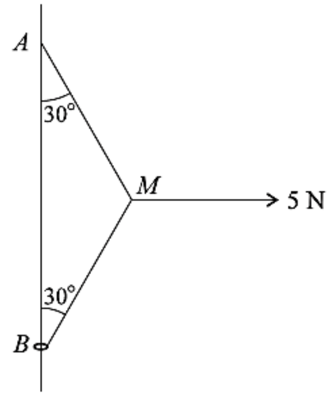
A light inextensible string has its ends attached to two fixed points A and B, with A vertically above B. A smooth ring R, of mass 0.8 kg, is threaded on the string and is pulled by a horizontal force of magnitude X newtons. The sections AR and BR of the string make angles of 50° and 20° respectively with the horizontal, as shown in the diagram. The ring rests in equilibrium with the string taut. Find

(i) the tension in the string, [3]

(ii) the value of X. [3]

Answers: **(i)** 18.9 N; **(ii)** 29.9. **N02/Q3**

14



One end of a light inextensible string is attached to a fixed point A of a fixed vertical wire. The other end of the string is attached to a small ring B, of mass 0.2 kg, through which the wire passes. A horizontal force of magnitude 5 N is applied to the mid-point M of the string. The system is in equilibrium with the string taut, with B below A, and with angles ABM and BAM equal to 30° (see diagram).

(i) Show that the tension in BM is 5 N. [3]

(ii) The ring is on the point of sliding up the wire. Find the coefficient of friction between the ring and the wire. [5]

(iii) A particle of mass m kg is attached to the ring. The ring is now on the point of sliding down the wire. Given that the coefficient of friction between the ring and the wire is unchanged, find the value of m. [2]

Answers: **(i)** 1.5 m, **(ii)** 1.05 m. **N03/Q6**

15

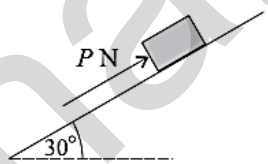


Fig. 1

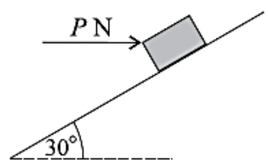


Fig. 2

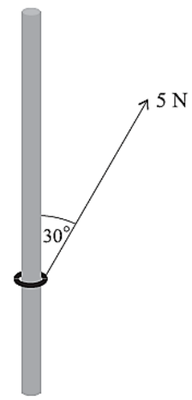
A small block of weight 18 N is held at rest on a smooth plane inclined at 30° to the horizontal, by a force of magnitude P N. Find

(i) the value of P when the force is parallel to the plane, as in Fig. 1, [2]

(ii) the value of P when the force is horizontal, as in Fig. 2. [3]

Answers: **(i)** $P = 9$; **(ii)** $P = 10.4$. **N04/Q2**

16



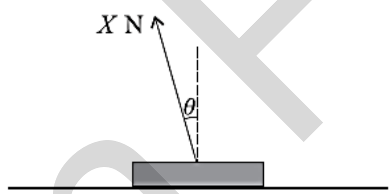
A small ring of mass 0.6 kg is threaded on a rough rod which is fixed vertically. The ring is in equilibrium, acted on by a force of magnitude 5 N pulling upwards at 30° to the vertical (see diagram).

(i) Show that the frictional force acting on the ring has magnitude 1.67 N, correct to 3 significant figures. [2]

(ii) The ring is on the point of sliding down the rod. Find the coefficient of friction between the ring and the rod. [3]

Answer: (ii) 0.668. **N06/Q2**

17



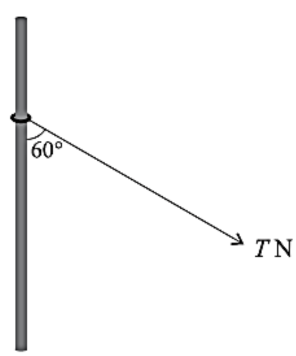
A stone slab of mass 320 kg rests in equilibrium on rough horizontal ground. A force of magnitude X N acts upwards on the slab at an angle of θ to the vertical, where $\tan\theta = \frac{7}{24}$ (see diagram).

(i) Find, in terms of X, the normal component of the force exerted on the slab by the ground. [3]

(ii) Given that the coefficient of friction between the slab and the ground is $\frac{3}{8}$, find the value of X for which the slab is about to slip. [3]

N05/Q4

18



A ring of mass 4 kg is threaded on a fixed rough vertical rod. A light string is attached to the ring, and is pulled with a force of magnitude T N acting at an angle of 60° to the downward vertical (see diagram). The ring is in equilibrium.

(i) The normal and frictional components of the contact force exerted on the ring by the rod are R N and F N respectively. Find R and F in terms of T. [4]

(ii) The coefficient of friction between the rod and the ring is 0.7. Find the value of T for which the ring is about to slip. [3]

Answers: (i) $R = T\sin 60°$, $F = 40 + T\cos 60°$; (ii) 377. **N07/Q5**

19

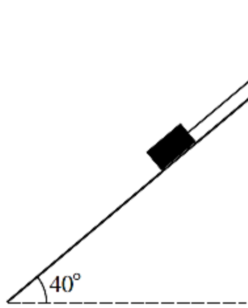


Fig. 1

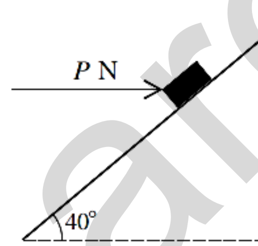


Fig. 2

A small block of weight 12 N is at rest on a smooth plane inclined at 40° to the horizontal. The block is held in equilibrium by a force of magnitude P N. Find the value of P when

(i) the force is parallel to the plane as in Fig. 1, [2]

(ii) the force is horizontal as in Fig. 2. [2]

Answers: (i) 7.71; (ii) 10.1. **N09/Q1**

20 A block of mass 400 kg rests in limiting equilibrium on horizontal ground. A force of magnitude 2000 N acts on the block at an angle of 15° to the upwards vertical. Find the coefficient of friction between the block and the ground, correct to 2 significant figures. [5]

N10/42/Q1

21 Two particles P and Q move on a line of greatest slope of a smooth inclined plane. The particles start at the same instant and from the same point, each with speed 1.3 m s^{-1}. Initially P moves down the plane and Q moves up the plane. The distance between the particles t seconds after they start to move is d m.

(i) Show that $d = 2.6t$. [4]

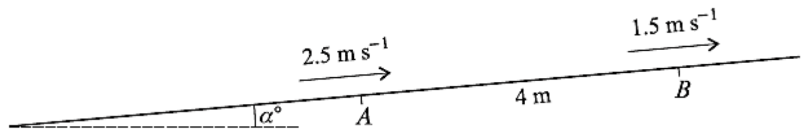
When $t = 2.5$ the difference in the vertical height of the particles is 1.6 m. Find

(ii) the acceleration of the particles down the plane, [3]

(iii) the distance travelled by P when Q is at its highest point. [3]

Answers: (ii) 2.46 ms^{-2}; (iii) 1.03 m. **J06/Q7**

22



A particle slides up a line of greatest slope of a smooth plane inclined at an angle $\alpha°$ to the horizontal. The particle passes through the points A and B with speeds $2.5\,\text{m s}^{-1}$ and $1.5\,\text{m s}^{-1}$ respectively. The distance AB is 4 m (see diagram). Find

(i) the deceleration of the particle, [2]

(ii) the value of α. [2]

Answers: (i) $0.5\ \text{ms}^{-2}$; (ii) 2.9. **J07/Q1**

23 A particle slides down a smooth plane inclined at an angle of $\alpha°$ to the horizontal. The particle passes through the point A with speed $1.5\,\text{m s}^{-1}$, and 1.2 s later it passes through the point B with speed $4.5\,\text{m s}^{-1}$. Find

(i) the acceleration of the particle, [2]

(ii) the value of α. [2]

Answers: (i) $2.5\ \text{ms}^{-2}$; (ii) 14.5. **J08/Q1**

24 A particle of mass m kg moves up a line of greatest slope of a rough plane inclined at 21° to the horizontal. The frictional and normal components of the contact force on the particle have magnitudes F N and R N respectively. The particle passes through the point P with speed $10\,\text{m s}^{-1}$, and 2 s later it reaches its highest point on the plane.

(i) Show that $R = 9.336m$ and $F = 1.416m$, each correct to 4 significant figures. [5]

(ii) Find the coefficient of friction between the particle and the plane. [1]

After the particle reaches its highest point it starts to move down the plane.

(iii) Find the speed with which the particle returns to P. [5]

Answers: (ii) 0.152; (iii) $6.58\ \text{ms}^{-1}$. **N06/Q7**

25 A particle moves up a line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\cos\alpha = 0.96$ and $\sin\alpha = 0.28$.

(i) Given that the normal component of the contact force acting on the particle has magnitude 1.2 N, find the mass of the particle. [2]

(ii) Given also that the frictional component of the contact force acting on the particle has magnitude 0.4 N, find the deceleration of the particle. [3]

The particle comes to rest on reaching the point X.

(iii) Determine whether the particle remains at X or whether it starts to move down the plane. [2]

Answers: (i) 0.125 kg; (ii) $6\ \text{ms}^{-2}$; (iii) Remains at X. **N09/Q4**

26

The velocity-time graph shown models the motion of a parachutist falling vertically. There are four stages in the motion:

- falling freely with the parachute closed,
- decelerating at a constant rate with the parachute open,
- falling with constant speed with the parachute open,
- coming to rest instantaneously on hitting the ground.

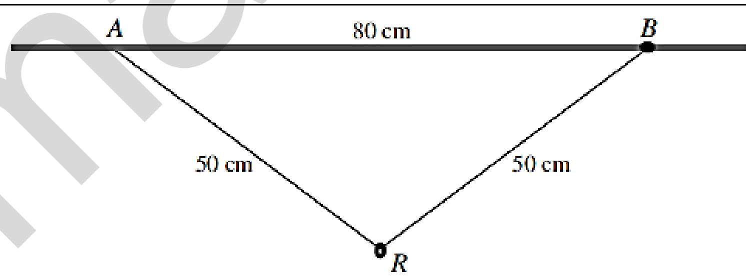
(i) Show that the total distance fallen is 1048 m. [2]

The weight of the parachutist is 850 N.

(ii) Find the upward force on the parachutist due to the parachute, during the second stage. [5]

Answer: (ii) 1360 N. **J11/42/Q3**

27



A small smooth ring R, of mass 0.6 kg, is threaded on a light inextensible string of length 100 cm. One end of the string is attached to a fixed point A. A small bead B of mass 0.4 kg is attached to the other end of the string, and is threaded on a fixed rough horizontal rod which passes through A. The system is in equilibrium with B at a distance of 80 cm from A (see diagram).

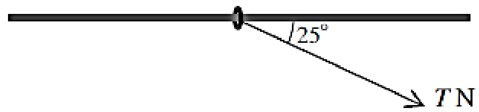
(i) Find the tension in the string. [3]

(ii) Find the frictional and normal components of the contact force acting on B. [4]

(iii) Given that the equilibrium is limiting, find the coefficient of friction between the bead and the rod. [2]

Answers: **(i)** 5 N; **(ii)** 4 N, 7 N; **(iii)** 0.571 **J11/42/Q6**

28



A ring of mass 4 kg is attached to one end of a light string. The ring is threaded on a fixed horizontal rod and the string is pulled at an angle of 25° below the horizontal (see diagram). With a tension in the string of T N the ring is in equilibrium.

(i) Find, in terms of T, the horizontal and vertical components of the force exerted on the ring by the rod. [4]

The coefficient of friction between the ring and the rod is 0.4.

(ii) Given that the equilibrium is limiting, find the value of T. [3]

Answers: **(i)** Horizontal $0.906T$, Vertical $40 + 0.423T$ **(ii)** $T = 21.7$ N **J12/42/Q4**

29 A string is attached to a block of weight 30 N, which is in contact with a rough horizontal plane. When the string is horizontal and the tension in it is 24 N, the block is in limiting equilibrium.

(i) Find the coefficient of friction between the block and the plane. [2]

The block is now in motion and the string is at an angle of 30° upwards from the plane. The tension in the string is 25 N.

(ii) Find the acceleration of the block. [4]

Answers: **(i)** Coefficient of friction $\mu = 0.8$ **(ii)** $a = 2.55$ ms^{-2} **J13/42/Q1**

30 A particle P is released from rest at the top of a smooth plane which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{16}{65}$. The distance travelled by P from the top to the bottom is S metres, and the speed of P at the bottom is 8 m s^{-1}.

(i) Find the value of S and hence find the speed of P when it has travelled $\frac{1}{2}S$ metres. [5]

The time taken by P to travel from the top to the bottom of the plane is T seconds.

(ii) Find the distance travelled by P at the instant when it has been moving for $\frac{1}{2}T$ seconds. [2]

Answers: **(i)** $S = 13$, speed = 5.66 ms^{-1} **(ii)** distance = 3.25 m **J13/42/Q4**

31 A particle P of mass 0.2 kg is released from rest at a point 7.2 m above the surface of the liquid in a container. P falls through the air and into the liquid. There is no air resistance and there is no instantaneous change of speed as P enters the liquid. When P is at a distance of 0.8 m below the surface of the liquid, P's speed is $6\ \mathrm{m\,s^{-1}}$. The only force on P due to the liquid is a constant resistance to motion of magnitude R N.

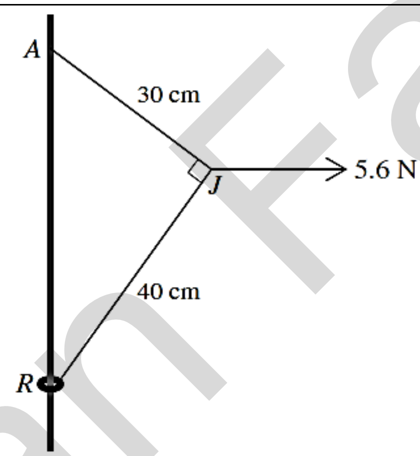
(i) Find the deceleration of P while it is falling through the liquid, and hence find the value of R. [5]

The depth of the liquid in the container is 3.6 m. P is taken from the container and attached to one end of a light inextensible string. P is placed at the bottom of the container and then pulled vertically upwards with constant acceleration. The resistance to motion of R N continues to act. The particle reaches the surface 4 s after leaving the bottom of the container.

(ii) Find the tension in the string. [4]

Answer: Deceleration = $67.5\ \mathrm{ms^{-2}}$ R = 15.5 *Answer*: Tension = 17.6 N **J14/42/Q6**

32



A small ring R is attached to one end of a light inextensible string of length 70 cm. A fixed rough vertical wire passes through the ring. The other end of the string is attached to a point A on the wire, vertically above R. A horizontal force of magnitude 5.6 N is applied to the point J of the string 30 cm from A and 40 cm from R. The system is in equilibrium with each of the parts AJ and JR of the string taut and angle AJR equal to 90° (see diagram).

(i) Find the tension in the part AJ of the string, and find the tension in the part JR of the string. [5]

The ring R has mass 0.2 kg and is in limiting equilibrium, on the point of moving up the wire.

(ii) Show that the coefficient of friction between R and the wire is 0.341, correct to 3 significant figures. [4]

A particle of mass m kg is attached to R and R is now in limiting equilibrium, on the point of moving down the wire.

(iii) Given that the coefficient of friction is unchanged, find the value of m. [3]

Answer: (i) Tension in AJ is 4.48 N Tension in JR is 3.36 N (ii) given μ = 0.341 (iii) m = 0.1376 (answers of either m = 0.137 or m = 0.138 were accepted) **J15/42/Q7**

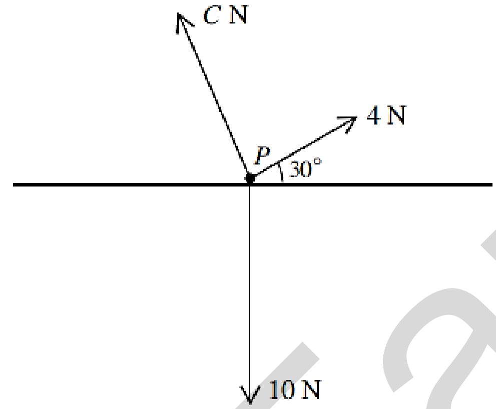
33 A block of mass 6 kg is sliding down a line of greatest slope of a plane inclined at 8° to the horizontal. The coefficient of friction between the block and the plane is 0.2.

(i) Find the deceleration of the block. [3]

(ii) Given that the initial speed of the block is $3\,\text{m}\,\text{s}^{-1}$, find how far the block travels. [2]

Answers: (i) $0.589\ \text{ms}^{-2}$; (ii) 7.64 m — **N11/42/Q2**

34



A particle P has weight 10 N and is in limiting equilibrium on a rough horizontal table. The forces shown in the diagram represent the weight of P, an applied force of magnitude 4 N acting on P in a direction at 30° above the horizontal, and the contact force exerted on P by the table (the resultant of the frictional and normal components) of magnitude C N.

(i) Find the value of C. [3]

(ii) Find the coefficient of friction between P and the table. [2]

Answers: (i) C = 8.72; (ii) 0.433 — **N11/42/Q4**

35 A, B and C are three points on a line of greatest slope of a plane which is inclined at $\theta°$ to the horizontal, with A higher than B and B higher than C. Between A and B the plane is smooth, and between B and C the plane is rough. A particle P is released from rest on the plane at A and slides down the line ABC. At time 0.8 s after leaving A, the particle passes through B with speed $4\,\text{m}\,\text{s}^{-1}$.

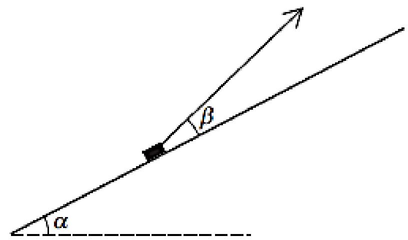
(i) Find the value of θ. [3]

At time 4.8 s after leaving A, the particle comes to rest at C.

(ii) Find the coefficient of friction between P and the rough part of the plane. [5]

Answers: (i) 30°; (ii) 0.693. — **N12/42/Q5**

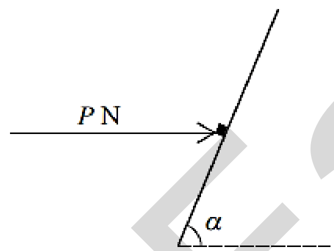
36



A small block of weight 5.1 N rests on a smooth plane inclined at an angle α to the horizontal, where $\sin\alpha = \frac{8}{17}$. The block is held in equilibrium by means of a light inextensible string. The string makes an angle β above the line of greatest slope on which the block rests, where $\sin\beta = \frac{7}{25}$ (see diagram). Find the tension in the string. [3]

Answer: $T = 2.5$ N N13/42/Q1

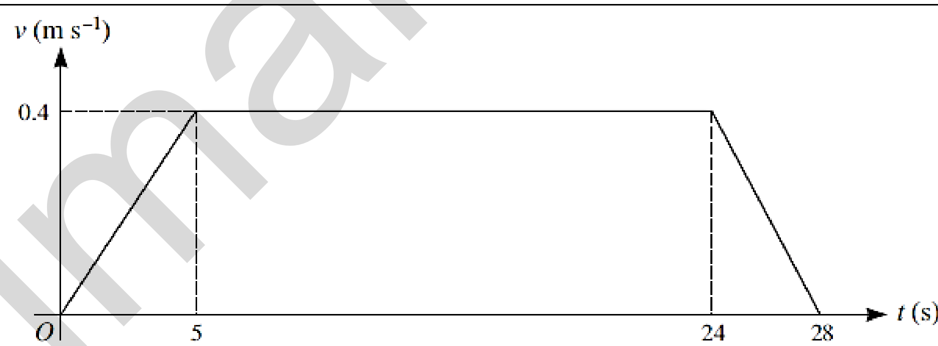
37



A rough plane is inclined at an angle α to the horizontal, where $\tan\alpha = 2.4$. A small block of mass 0.6 kg is held at rest on the plane by a horizontal force of magnitude P N. This force acts in a vertical plane through a line of greatest slope (see diagram). The coefficient of friction between the block and the plane is 0.4. The block is on the point of slipping down the plane. By resolving forces parallel to and perpendicular to the inclined plane, or otherwise, find the value of P. [8]

Answer: $P = 6.12$ N13/42/Q4

38



An elevator is pulled vertically upwards by a cable. The velocity-time graph for the motion is shown above. Find

(i) the distance travelled by the elevator, [2]

(ii) the acceleration during the first stage and the deceleration during the third stage. [2]

The mass of the elevator is 800 kg and there is a box of mass 100 kg on the floor of the elevator.

(iii) Find the tension in the cable in each of the three stages of the motion. [3]

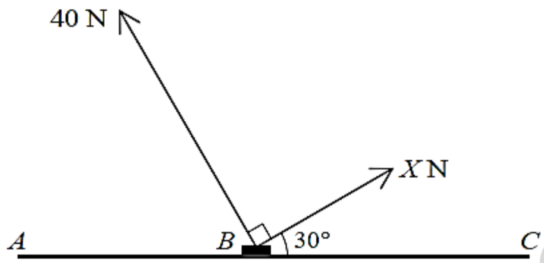
(iv) Find the greatest and least values of the magnitude of the force exerted on the box by the floor of the elevator. [3]

Answer: 9.4 m *Answer*: 0.08 ms^{-2} and 0.1 ms^{-2} **N13/42/Q7**

Answer: Stage 1: T = 9072 N, Stage 2: T = 9000 N, Stage 3 T = 8910 N

Answer: Greatest reaction force = 1008 N, Least reaction force = 990 N

39



Forces of magnitude X N and 40 N act on a block B of mass 15 kg, which is in equilibrium in contact with a horizontal surface between points A and C on the surface. The forces act in the same vertical plane and in the directions shown in the diagram.

(i) Given that the surface is smooth, find the value of X. [2]

(ii) It is given instead that the surface is rough and that the block is in limiting equilibrium. The frictional force acting on the block has magnitude 10 N in the direction towards A. Find the coefficient of friction between the block and the surface. [5]

Answer: 23.1, 0.102 **N14/42/Q4**

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40 ABC is a line of greatest slope of a plane inclined at angle α to the horizontal, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$. The point A is at the top of the plane, the point C is at the bottom of the plane and the length of AC is 5 m. The part of the plane above the level of B is smooth and the part below the level of B is rough. A particle P is released from rest at A and reaches C with a speed of 2 m s^{-1}. The coefficient of friction between P and the part of the plane below B is 0.5. Find

(i) the acceleration of P while moving

(a) from A to B,

(b) from B to C, [3]

(ii) the distance AB, [3]

(iii) the time taken for P to move from A to C. [3]

Answer: 2.8, -2, 2.5, 2.21 **N14/42/Q6**

41

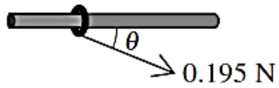

Fig. 1

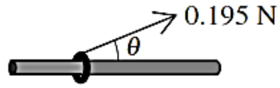

Fig. 2

A small ring of mass 0.024 kg is threaded on a fixed rough horizontal rod. A light inextensible string is attached to the ring and the string is pulled with a force of magnitude 0.195 N at an angle of θ with the horizontal, where $\sin\theta = \frac{5}{13}$. When the angle θ is below the horizontal (see Fig. 1) the ring is in limiting equilibrium.

(i) Find the coefficient of friction between the ring and the rod. [6]

When the angle θ is above the horizontal (see Fig. 2) the ring moves.

(ii) Find the acceleration of the ring. [4]

Answer: The coefficient of friction between the ring and the rod is $\mu = \frac{4}{7} = 0.571$ N15/42/Q6

Answers: Acceleration of the ring $= \frac{25}{7} = 3.57\ ms^{-2}$

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42 A block of mass 20 kg is at rest on a plane inclined at 10° to the horizontal. A force acts on the block parallel to a line of greatest slope of the plane. The coefficient of friction between the block and the plane is 0.32. Find the least magnitude of the force necessary to move the block,

(i) given that the force acts up the plane,

(ii) given instead that the force acts down the plane.

[6]

Answers: (i) 97.8 N; (ii) 28.3 N. N08/Q2

HOMEWORK: LAWS OF MOTION VARIANT 41 & 43

1 A small box of mass 5 kg is pulled at a constant speed of $2.5\,\text{m s}^{-1}$ down a line of greatest slope of a rough plane inclined at $10°$ to the horizontal. The pulling force has magnitude 20 N and acts downwards parallel to a line of greatest slope of the plane.

(i) Find the coefficient of friction between the box and the plane. [5]

The pulling force is removed while the box is moving at $2.5\,\text{m s}^{-1}$.

(ii) Find the distance moved by the box after the instant at which the pulling force is removed. [4]

Answer: **(i)** $\mu = 0.582$ **(ii)** Distance moved by the box = 0.781 m — 43/J15/6

2 A block B of mass 2.7 kg is pulled at constant speed along a straight line on a rough horizontal floor. The pulling force has magnitude 25 N and acts at an angle of θ above the horizontal. The normal component of the contact force acting on B has magnitude 20 N.

(i) Show that $\sin\theta = 0.28$. [2]

(ii) Find the work done by the pulling force in moving the block a distance of 5 m. [2]

Answer: **(ii)** Work done by the pulling force is 120 J — 41/J15/1

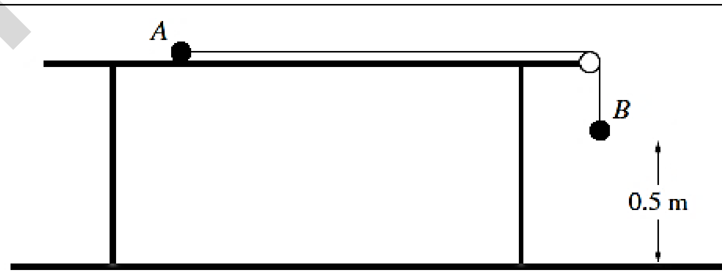
3 A block of weight 6.1 N slides down a slope inclined at $\tan^{-1}\left(\frac{11}{60}\right)$ to the horizontal. The coefficient of friction between the block and the slope is $\frac{1}{4}$. The block passes through a point A with speed $2\,\text{m s}^{-1}$. Find how far the block moves from A before it comes to rest. [5]

Answer: Distance moved by the block from A before it comes to rest is 3.05 m — 41/J15/3

4 A cyclist and her bicycle have a total mass of 84 kg. She works at a constant rate of P W while moving on a straight road which is inclined to the horizontal at an angle θ, where $\sin\theta = 0.1$. When moving uphill, the cyclist's acceleration is $1.25\,\text{m s}^{-2}$ at an instant when her speed is $3\,\text{m s}^{-1}$. When moving downhill, the cyclist's acceleration is $1.25\,\text{m s}^{-2}$ at an instant when her speed is $10\,\text{m s}^{-1}$. The resistance to the cyclist's motion, whether the cyclist is moving uphill or downhill, is R N. Find the values of P and R. [8]

- *Answer:* $P = 720$ $R = 51$ — 41/J15/5

5



Particles A and B, of masses 0.3 kg and 0.7 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest on a rough horizontal table with the string passing over a smooth pulley fixed at the edge of the table. The coefficient of friction between A and the table is 0.2. Particle B hangs vertically below the pulley at a height of 0.5 m above the floor (see diagram). The system is released from rest and 0.25 s later the string breaks. A does not reach the pulley in the subsequent motion. Find

(i) the speed of B immediately before it hits the floor, [9]

(ii) the total distance travelled by A. [3]

Answer: **(i)** Speed of B immediately before it hits the floor = 2.93 ms^{-1}
(ii) Total distance travelled by A is 0.84 m

41/J15/7

6 A car of mass 1400 kg moves on a horizontal straight road. The resistance to the car's motion is constant and equal to 800 N and the power of the car's engine is constant and equal to P W. At an instant when the car's speed is 18 m s^{-1} its acceleration is 0.5 m s^{-2}.

(i) Find the value of P. [3]

The car continues and passes through another point with speed 25 m s^{-1}.

(ii) Find the car's acceleration at this point. [2]

Answer: 27000, 0.2

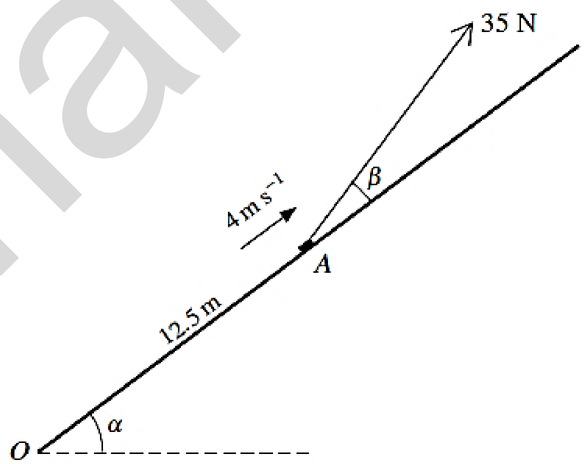
43/N14/1

7 A box of mass 8 kg is on a rough plane inclined at 5° to the horizontal. A force of magnitude P N acts on the box in a direction upwards and parallel to a line of greatest slope of the plane. When $P = 7X$ the box moves up the line of greatest slope with acceleration 0.15 m s^{-2} and when $P = 8X$ the box moves up the line of greatest slope with acceleration 1.15 m s^{-2}. Find the value of X and the coefficient of friction between the box and the plane. [8]

Answer: 8, 0.600

43/N14/5

8



A small block of mass 3 kg is initially at rest at the bottom O of a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.6$ and $\cos\alpha = 0.8$. A force of magnitude 35 N acts on the block at an angle β above the plane, where $\sin\beta = 0.28$ and $\cos\beta = 0.96$. The block starts to move up a line of greatest slope of the plane and passes through a point A with speed 4 m s^{-1}. The distance OA is 12.5 m (see diagram).

(i) For the motion of the block from O to A, find the work done against the frictional force acting on the block. [4]

(ii) Find the coefficient of friction between the block and the plane. [3]

At the instant that the block passes through A the force of magnitude 35 N ceases to act.

(iii) Find the distance the block travels up the plane after passing through A. [4]

Answer: 171, 0.963, 0.584 43/N14/7

9 A car of mass 800 kg is moving on a straight horizontal road with its engine working at a rate of 22.5 kW. Find the resistance to the car's motion at an instant when the car's speed is $18\,\text{m s}^{-1}$ and its acceleration is $1.2\,\text{m s}^{-2}$. [4]

Answer: 290 41/N14/1

10

Fig. 1 **Fig. 2**

A block of weight 7.5 N is at rest on a plane which is inclined to the horizontal at angle α, where $\tan\alpha = \frac{7}{24}$. The coefficient of friction between the block and the plane is μ. A force of magnitude 7.2 N acting parallel to a line of greatest slope is applied to the block. When the force acts up the plane (see Fig. 1) the block remains at rest.

(i) Show that $\mu \geqslant \frac{17}{24}$. [4]

When the force acts down the plane (see Fig. 2) the block slides downwards.

(ii) Show that $\mu < \frac{31}{24}$. [2]

Answer: 41/N14/3

11 A car of mass 1250 kg travels up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = 0.02$. The power provided by the car's engine is 23 kW. The resistance to motion is constant and equal to 600 N. Find the speed of the car at an instant when its acceleration is $0.5\,\text{m s}^{-2}$. [5]

Answer: $15.6\,\text{ms}^{-1}$ 43/J14/2

12 A particle moves up a line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.28$. The coefficient of friction between the particle and the plane is $\frac{1}{3}$.

(i) Show that the acceleration of the particle is $-6\,\text{m s}^{-2}$. [3]

(ii) Given that the particle's initial speed is $5.4\,\text{m s}^{-1}$, find the distance that the particle travels up the plane. [2]

Answer: 2.43 m 43/N13/1

13 A box of mass 30 kg is at rest on a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.1$, acted on by a force of magnitude 40 N. The force acts upwards and parallel to a line of greatest slope of the plane. The box is on the point of slipping up the plane.

(i) Find the coefficient of friction between the box and the plane. [5]

The force of magnitude 40 N is removed.

(ii) Determine, giving a reason, whether or not the box remains in equilibrium. [2]

Answer: 0.0335 Not in equilibrium 43/N13/4

14 A cyclist exerts a constant driving force of magnitude F N while moving up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{36}{325}$. A constant resistance to motion of 32 N acts on the cyclist. The total weight of the cyclist and his bicycle is 780 N. The cyclist's acceleration is $-0.2\ \text{m s}^{-2}$.

(i) Find the value of F. [4]

The cyclist's speed is $7\ \text{m s}^{-1}$ at the bottom of the hill.

(ii) Find how far up the hill the cyclist travels before coming to rest. [2]

Answer: (i) $F = 103$ (ii) 122.5 m 41/N13/3

15 Particles P and Q are moving in a straight line on a rough horizontal plane. The frictional forces are the only horizontal forces acting on the particles.

(i) Find the deceleration of each of the particles given that the coefficient of friction between P and the plane is 0.2, and between Q and the plane is 0.25. [2]

At a certain instant, P passes through the point A and Q passes through the point B. The distance AB is 5 m. The velocities of P and Q at A and B are $8\ \text{m s}^{-1}$ and $3\ \text{m s}^{-1}$, respectively, both in the direction AB.

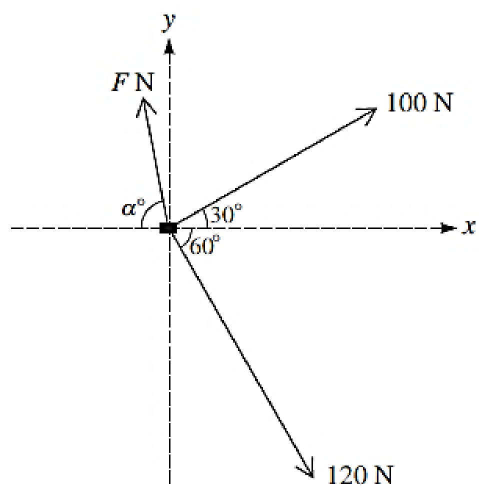
(ii) Find the speeds of P and Q immediately before they collide. [5]

Answer: (i) $2\ \text{ms}^{-2}$, 2.5ms^{-2} (ii) $6.09\ \text{ms}^{-1}$, $0.614\ \text{ms}^{-1}$ 41/N13/4

16 A straight ice track of length 50 m is inclined at 14° to the horizontal. A man starts at the top of the track, on a sledge, with speed $8\ \text{m s}^{-1}$. He travels on the sledge to the bottom of the track. The coefficient of friction between the sledge and the track is 0.02. Find the speed of the sledge and the man when they reach the bottom of the track. [4]

Answer: $16.9\ \text{ms}^{-1}$ 43/J13/1

17



A small box of mass 40 kg is moved along a rough horizontal floor by three men. Two of the men apply horizontal forces of magnitudes 100 N and 120 N, making angles of 30° and 60° respectively with the positive x-direction. The third man applies a horizontal force of magnitude F N making an angle of $\alpha°$ with the negative x-direction (see diagram). The resultant of the three horizontal forces acting on the box is in the positive x-direction and has magnitude 136 N.

(i) Find the values of F and α. [6]

(ii) Given that the box is moving with constant speed, state the magnitude of the frictional force acting on the box and hence find the coefficient of friction between the box and the floor. [3]

Answer. F=55.0; α=78.9; 0.34 43/J13/6

18 A block is at rest on a rough horizontal plane. The coefficient of friction between the block and the plane is 1.25.

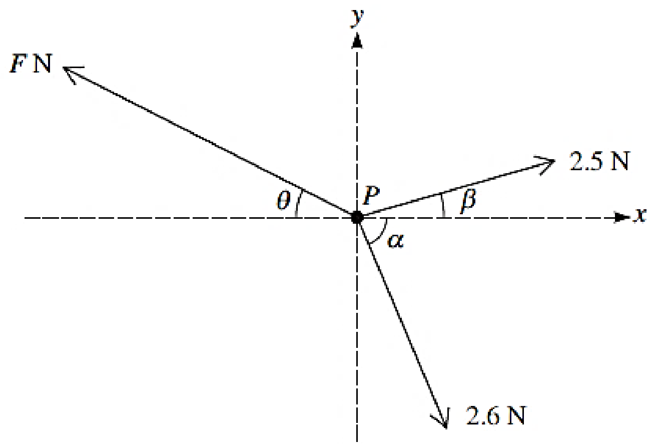
(i) State, giving a reason for your answer, whether the minimum vertical force required to move the block is greater or less than the minimum horizontal force required to move the block. [2]

A horizontal force of continuously increasing magnitude P N and fixed direction is applied to the block.

(ii) Given that the weight of the block is 60 N, find the value of P when the acceleration of the block is $4\,\text{m}\,\text{s}^{-2}$. [2]

41/J13/1

19



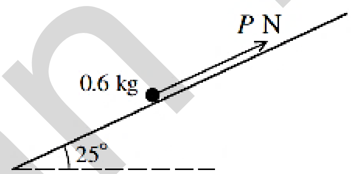
A particle P of mass 0.5 kg lies on a smooth horizontal plane. Horizontal forces of magnitudes F N, 2.5 N and 2.6 N act on P. The directions of the forces are as shown in the diagram, where $\tan\alpha = \frac{12}{5}$ and $\tan\beta = \frac{7}{24}$.

(i) Given that P is in equilibrium, find the values of F and $\tan\theta$. [6]

(ii) The force of magnitude F N is removed. Find the magnitude and direction of the acceleration with which P starts to move. [3]

Answer: i) 3.80 and 0.5 ii) 7.6 at 26.6 degrees from the positive x axis 41/J13/6

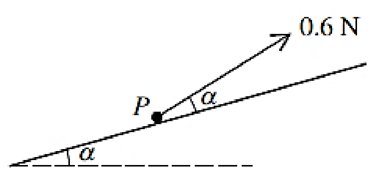
20



The diagram shows a particle of mass 0.6 kg on a plane inclined at 25° to the horizontal. The particle is acted on by a force of magnitude P N directed up the plane parallel to a line of greatest slope. The coefficient of friction between the particle and the plane is 0.36. Given that the particle is in equilibrium, find the set of possible values of P. [9]

Answer. $0.578 \le P \le 4.49$. 43/N12/6

21



A particle P of mass 0.5 kg rests on a rough plane inclined at angle α to the horizontal, where $\sin\alpha = 0.28$. A force of magnitude 0.6 N, acting upwards on P at angle α from a line of greatest slope of the plane, is just sufficient to prevent P sliding down the plane (see diagram). Find

(i) the normal component of the contact force on P, [2]

(ii) the frictional component of the contact force on P, [3]

(iii) the coefficient of friction between P and the plane. [2]

Answers: (i) 4.63 N; (ii) 0.824 N; (iii) 0.178. 41/N12/3

22

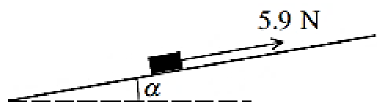


Fig. 1

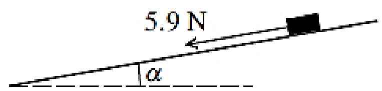


Fig. 2

A block of weight 6.1 N is at rest on a plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{11}{60}$. The coefficient of friction between the block and the plane is μ. A force of magnitude 5.9 N acting parallel to a line of greatest slope is applied to the block.

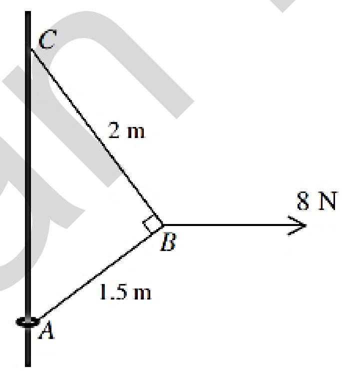
(i) When the force acts up the plane (see Fig. 1) the block remains at rest. Show that $\mu \geqslant \frac{4}{5}$. [5]

(ii) When the force acts down the plane (see Fig. 2) the block slides downwards. Show that $\mu < \frac{7}{6}$. [2]

(iii) Given that the acceleration of the block is 1.7 m s^{-2} when the force acts down the plane, find the value of μ. [2]

Answer. $\mu = 0.994$ 43/J12/6

23



A small ring of mass 0.2 kg is threaded on a fixed vertical rod. The end A of a light inextensible string is attached to the ring. The other end C of the string is attached to a fixed point of the rod above A. A horizontal force of magnitude 8 N is applied to the point B of the string, where $AB = 1.5$ m and $BC = 2$ m. The system is in equilibrium with the string taut and AB at right angles to BC (see diagram).

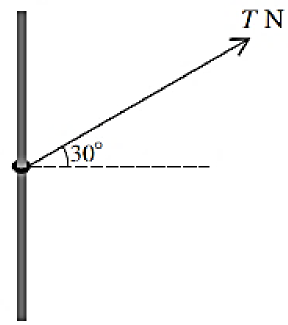
(i) Find the tension in the part AB of the string and the tension in the part BC of the string. [5]

The equilibrium is limiting with the ring on the point of sliding up the rod.

(ii) Find the coefficient of friction between the ring and the rod. [5]

Answers. **(i)** Tension in AB = 6.4 N, tension in BC = 4.8 N **(ii)** 0.359 41/J12/7

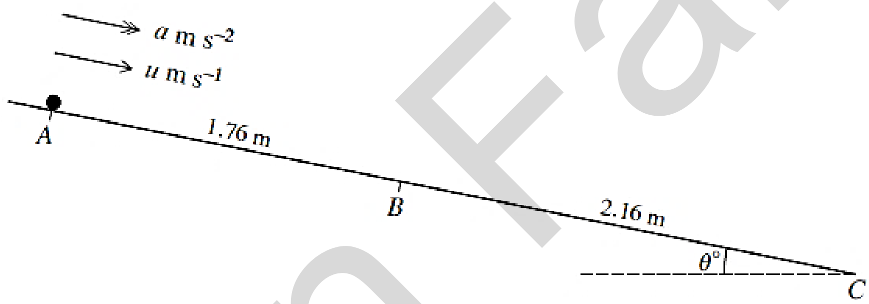
24



The diagram shows a ring of mass 2 kg threaded on a fixed rough vertical rod. A light string is attached to the ring and is pulled upwards at an angle of 30° to the horizontal. The tension in the string is T N. The coefficient of friction between the ring and the rod is 0.24. Find the two values of T for which the ring is in limiting equilibrium. [8]

Answers: 28.3, 68.5 43/N11/6

25



A, B and C are three points on a line of greatest slope of a smooth plane inclined at an angle of $\theta°$ to the horizontal. A is higher than B and B is higher than C, and the distances AB and BC are 1.76 m and 2.16 m respectively. A particle slides down the plane with constant acceleration a m s^{-2}. The speed of the particle at A is u m s^{-1} (see diagram). The particle takes 0.8 s to travel from A to B and takes 1.4 s to travel from A to C. Find

(i) the values of u and a, [6]

(ii) the value of θ. [2]

Answers: **(i)** $u = 1.4$, $a = 2$; **(ii)** $\theta = 11.5°$ 41/N11/4

26

Fig. 1

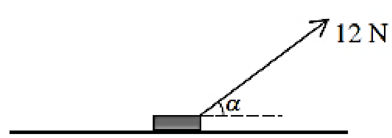


Fig. 2

A block of mass 2 kg is at rest on a horizontal floor. The coefficient of friction between the block and the floor is μ. A force of magnitude 12 N acts on the block at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. When the applied force acts downwards as in Fig. 1 the block remains at rest.

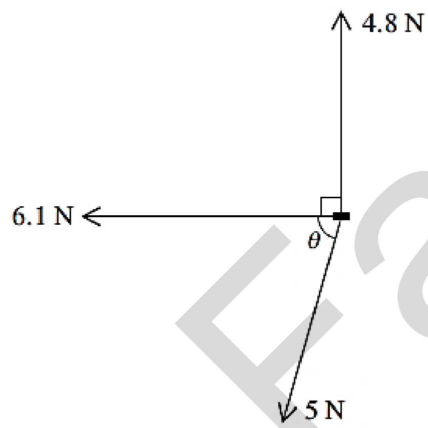
(i) Show that $\mu \geqslant \frac{6}{17}$. [5]

When the applied force acts upwards as in Fig. 2 the block slides along the floor.

(ii) Find another inequality for μ. [3]

Answer. (ii) $\mu < 3/4$ 41/N11/5

27



A small block of mass 1.25 kg is on a horizontal surface. Three horizontal forces, with magnitudes and directions as shown in the diagram, are applied to the block. The angle θ is such that $\cos\theta = 0.28$ and $\sin\theta = 0.96$. A horizontal frictional force also acts on the block, and the block is in equilibrium.

(i) Show that the magnitude of the frictional force is 7.5 N and state the direction of this force. [4]

(ii) Given that the block is in limiting equilibrium, find the coefficient of friction between the block and the surface. [2]

The force of magnitude 6.1 N is now replaced by a force of magnitude 8.6 N acting in the same direction, and the block begins to move.

(iii) Find the magnitude and direction of the acceleration of the block. [3]

Answers. (i) Opposite in direction to the force of magnitude 6.1 N; (ii) 0.6; (iii) 2 ms^{-2} in the direction of the force of magnitude 6.1 N. 43/J11/5

28 A block of mass 11 kg is at rest on a rough plane inclined at 30° to the horizontal. A force acts on the block in a direction up the plane parallel to a line of greatest slope. When the magnitude of the force is $2X$ N the block is on the point of sliding down the plane, and when the magnitude of the force is $9X$ N the block is on the point of sliding up the plane. Find

(i) the value of X, [3]

(ii) the coefficient of friction between the block and the plane. [4]

Answers. (i) 10; (ii) 0.367. 41/J11/4

29 A particle P is released from rest at a point on a smooth plane inclined at 30° to the horizontal. Find the speed of P

(i) when it has travelled 0.9 m,

(ii) 0.8 s after it is released.

[4]

Answers: (i) 3 ms^{-1}; (ii) 4 ms^{-1}. 43/N10/1

30 Particles P and Q move on a line of greatest slope of a smooth inclined plane. P is released from rest at a point O on the line and 2 s later passes through the point A with speed 3.5 $m\,s^{-1}$.

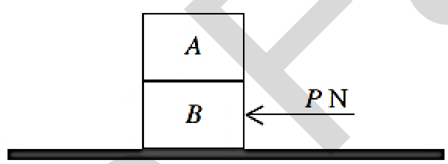
(i) Find the acceleration of P and the angle of inclination of the plane. [4]

At the instant that P passes through A the particle Q is released from rest at O. At time t s after Q is released from O, the particles P and Q are 4.9 m apart.

(ii) Find the value of t. [5]

Answers: (i) 1.75 ms^{-2}, 10.1°; (ii) 0.4. 43/J10/6

31



Two rectangular boxes A and B are of identical size. The boxes are at rest on a rough horizontal floor with A on top of B. Box A has mass 200 kg and box B has mass 250 kg. A horizontal force of magnitude P N is applied to B (see diagram). The boxes remain at rest if $P \leqslant 3150$ and start to move if $P > 3150$.

(i) Find the coefficient of friction between B and the floor. [3]

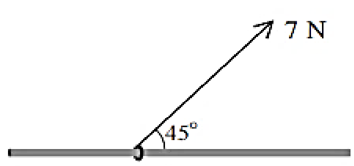
The coefficient of friction between the two boxes is 0.2. Given that $P > 3150$ and that no sliding takes place between the boxes,

(ii) show that the acceleration of the boxes is not greater than 2 $m\,s^{-2}$, [3]

(iii) find the maximum possible value of P. [3]

Answers: (i) 0.7; (iii) 4050. 43/J10/7

32



A small ring of mass 0.8 kg is threaded on a rough rod which is fixed horizontally. The ring is in equilibrium, acted on by a force of magnitude 7 N pulling upwards at 45° to the horizontal (see diagram).

(i) Show that the normal component of the contact force acting on the ring has magnitude 3.05 N, correct to 3 significant figures. [2]

(ii) The ring is in limiting equilibrium. Find the coefficient of friction between the ring and the rod. [3]

Answer: ii) 1.62	41/J10/3

33 A particle of mass 15 kg is stationary on a rough plane inclined at an angle of 20° to the horizontal. The coefficient of friction between the particle and the plane is 0.2. A force of magnitude X N acting parallel to a line of greatest slope of the plane is used to keep the particle in equilibrium. Show that the least possible value of X is 23.1, correct to 3 significant figures, and find the greatest possible value of X. [7]

Answer: 79.5	J16/43/Q4

34 A particle of mass 0.8 kg is projected with a speed of $12\,\mathrm{m\,s^{-1}}$ up a line of greatest slope of a rough plane inclined at an angle of 10° to the horizontal. The coefficient of friction between the particle and the plane is 0.4.

(i) Find the acceleration of the particle. [4]

(ii) Find the distance the particle moves up the plane before coming to rest. [2]

Answers: (i) The acceleration of the particle is $-5.68\,\mathrm{ms^{-2}}$ (to three significant figures) (ii) Distance moved up the plane before coming to rest is 12.7 m	J17/41/Q2

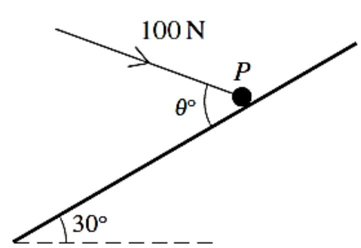
35 A man pushes a wheelbarrow of mass 25 kg along a horizontal road with a constant force of magnitude 35 N at an angle of 20° below the horizontal. There is a constant resistance to motion of 15 N. The wheelbarrow moves a distance of 12 m from rest.

(i) Find the work done by the man. [2]

(ii) Find the speed attained by the wheelbarrow after 12 m. [3]

Answers: (i) 395 J (ii) $4.14\,\mathrm{ms^{-1}}$	J17/43/Q1

36



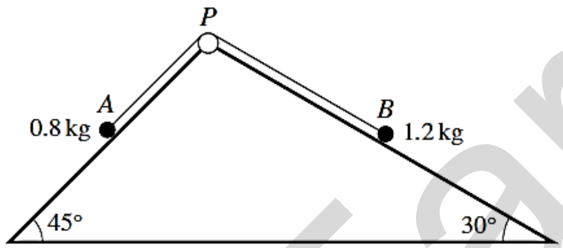
A particle P of mass 8 kg is on a smooth plane inclined at an angle of 30° to the horizontal. A force of magnitude 100 N, making an angle of $\theta°$ with a line of greatest slope and lying in the vertical plane containing the line of greatest slope, acts on P (see diagram).

(i) Given that P is in equilibrium, show that $\theta = 66.4$, correct to 1 decimal place, and find the normal reaction between the plane and P. [4]

(ii) Given instead that $\theta = 30$, find the acceleration of P. [2]

Answers: (i) $\theta = 66.4$ (given) The normal reaction $R = 161$ N (to 3 significant figures
(ii) The acceleration of P is 5.83 ms^{-2} J18/41/Q3

37



The diagram shows a triangular block with sloping faces inclined to the horizontal at 45° and 30°. Particle A of mass 0.8 kg lies on the face inclined at 45° and particle B of mass 1.2 kg lies on the face inclined at 30°. The particles are connected by a light inextensible string which passes over a small smooth pulley P fixed at the top of the faces. The parts AP and BP of the string are parallel to lines of greatest slope of the respective faces. The particles are released from rest with both parts of the string taut. In the subsequent motion neither particle reaches the pulley and neither particle reaches the bottom of a face.

(i) Given that both faces are smooth, find the speed of A after each particle has travelled a distance of 0.4 m. [6]

(ii) It is given instead that both faces are rough. The coefficient of friction between each particle and a face of the block is μ. Find the value of μ for which the system is in limiting equilibrium. [6]

Answers: (i) The speed of A after each particle has travelled a distance 0.4 m is 0.370 ms–1 (to 3 sf)
(ii) The value of μ for which the system is in limiting equilibrium is 0.0214 (to 3 sf) J18/41/Q7

38 A particle of mass 3 kg is on a rough plane inclined at an angle of 20° to the horizontal. A force of magnitude P N acting parallel to a line of greatest slope of the plane is used to keep the particle in equilibrium. The coefficient of friction between the particle and the plane is 0.35. Show that the least possible value of P is 0.394, correct to 3 significant figures, and find the greatest possible value of P. [6]

Answer: 20.1 J18/43/Q5

39 A particle of mass 0.1 kg is released from rest on a rough plane inclined at 20° to the horizontal. It is given that, 5 seconds after release, the particle has a speed of $2\,\text{m}\,\text{s}^{-1}$.

(i) Find the acceleration of the particle and hence show that the magnitude of the frictional force acting on the particle is 0.302 N, correct to 3 significant figures. [3]

(ii) Find the coefficient of friction between the particle and the plane. [2]

Answers: The acceleration of the particle is $0.4\,\text{ms}^{-2}$ Frictional force = 0.302 N (Given) N16/41/Q2

Answer: The coefficient of friction between the particle and the plane is 0.321

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40 A box of mass 50 kg is at rest on a plane inclined at 10° to the horizontal.

(i) Find an inequality for the coefficient of friction between the box and the plane. [2]

In fact the coefficient of friction between the box and the plane is 0.19.

(ii) A girl pushes the box with a force of 50 N, acting down a line of greatest slope of the plane, for a distance of 5 m. She then stops pushing. Use an energy method to find the speed of the box when it has travelled a further 5 m. [5]

The box then comes to a plane inclined at 20° below the horizontal. The box moves down a line of greatest slope of this plane. The coefficient of friction is still 0.19 and the girl is not pushing the box.

(iii) Find the acceleration of the box. [2]

Answer: □ $\geqslant$ 0.176 N16/43/Q7

Answer: The speed of the box after it has travelled a further 5 m is $2.70\,\text{ms}^{-1}$

Answer: The acceleration of the box is $1.63\,\text{ms}^{-2}$

---

41 A block of mass 3 kg is initially at rest on a smooth horizontal floor. A force of 12 N, acting at an angle of 25° above the horizontal, is applied to the block. Find the distance travelled by the block in the first 5 seconds of its motion. [4]

Answer: The power output of the tractor's engine is 13 800 W = 13.8 kW N17/41/Q1

Answer: The acceleration of the tractor as it begins to climb the hill is $-0.445\,\text{m}\,\text{s}^{-2}$ (to 3sf)

Answer: The constant speed that the tractor can maintain on the hill is $6.70\,\text{m}\,\text{s}^{-1}$ (to 3sf)

---

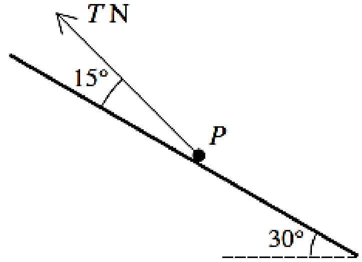
42 A particle is released from rest and slides down a line of greatest slope of a rough plane which is inclined at 25° to the horizontal. The coefficient of friction between the particle and the plane is 0.4.

(i) Find the acceleration of the particle. [4]

(ii) Find the distance travelled by the particle in the first 3 s after it is released. [2]

Answers: (i) $0.601\,\text{ms}^{-2}$ (ii) 2.70 m N17/43/Q3

43



A particle P of mass 0.2 kg rests on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the particle and the plane is 0.3. A force of magnitude T N acts upwards on P at 15° above a line of greatest slope of the plane (see diagram).

(i) Find the least value of T for which the particle remains at rest. [6]

The force of magnitude T N is now removed. A new force of magnitude 0.25 N acts on P up the plane, parallel to a line of greatest slope of the plane. Starting from rest, P slides down the plane. After moving a distance of 3 m, P passes through the point A.

(ii) Use an energy method to find the speed of P at A. [5]

Answers: (i) 0.541 (ii) 2.63 ms^{-1} N17/43/Q7

44 A particle of mass 0.2 kg moving in a straight line experiences a constant resistance force of 1.5 N. When the particle is moving at speed 2.5 m s^{-1}, a constant force of magnitude F N is applied to it in the direction in which it is moving. Given that the speed of the particle 5 seconds later is 4.5 m s^{-1}, find the value of F. [4]

Answer: F = 1.58 N18/41/Q1

45 A particle is projected from a point P with initial speed u m s^{-1} up a line of greatest slope PQR of a rough inclined plane. The distances PQ and QR are both equal to 0.8 m. The particle takes 0.6 s to travel from P to Q and 1 s to travel from Q to R.

(i) Show that the deceleration of the particle is $\frac{2}{3}$ m s^{-2} and hence find u, giving your answer as an exact fraction. [6]

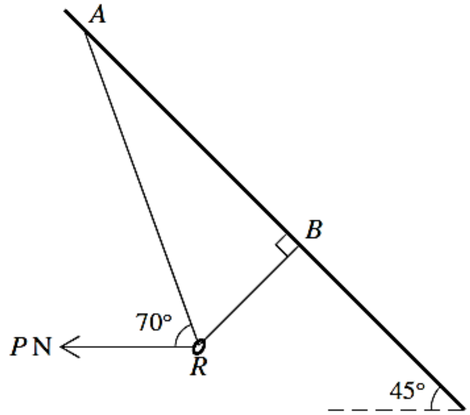
(ii) Given that the plane is inclined at 3° to the horizontal, find the value of the coefficient of friction between the particle and the plane. [4]

N18/41/Q6

Answers: (i) $a = -\dfrac{2}{3}$ m s^{-2} (Answer given) $u = \dfrac{23}{15}$

(ii) □= 0.0144 (to 3 sf)

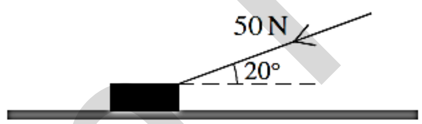
46



A small smooth ring R of mass 0.2 kg is threaded onto a light inextensible string ARB. The two ends of the string are attached to points A and B on a sloping roof inclined at 45° to the horizontal. A horizontal force of magnitude P N, acting in the plane ARB, is applied to the ring. The section BR of the string is perpendicular to the roof and the section AR of the string is inclined at 70° to the horizontal (see diagram). The system is in equilibrium. Find the tension in the string and the value of P. [4]

Answer: Tension in string = 1.21 N P = 0.443 N18/43/Q1

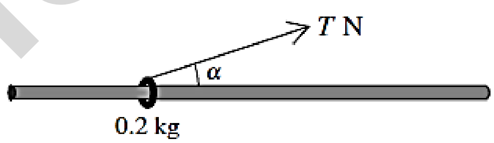
47



A block is pushed along a horizontal floor by a force of magnitude 50 N which acts at an angle of 20° to the horizontal (see diagram). The coefficient of friction between the block and the floor is 0.3. Given that the speed of the block is constant, find the mass of the block. [5]

Answer: 14.0 kg N18/43/Q2

48



A ring of mass 0.2 kg is threaded on a fixed rough horizontal rod and a light inextensible string is attached to the ring at an angle α above the horizontal, where $\cos\alpha = 0.96$. The ring is in limiting equilibrium with the tension in the string T N (see diagram). Given that the coefficient of friction between the ring and the rod is 0.25, find the value of T. [5]

Answer: 0.485 N N15/43/Q2

49 A particle of mass 0.5 kg starts from rest and slides down a line of greatest slope of a smooth plane. The plane is inclined at an angle of 30° to the horizontal.

(i) Find the time taken for the particle to reach a speed of 2.5 m s^{-1}. [3]

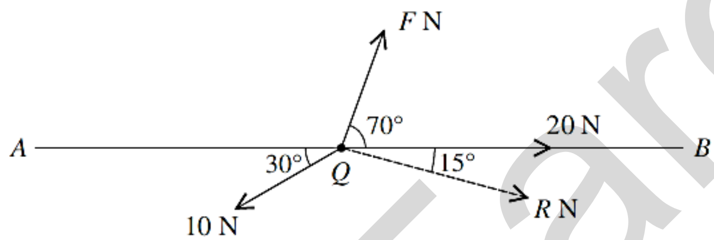
When the particle has travelled 3 m down the slope from its starting point, it reaches rough horizontal ground at the bottom of the slope. The frictional force acting on the particle is 1 N.

(ii) Find the distance that the particle travels along the ground before it comes to rest. [3]

Answer: Time taken for the particle to reach a speed of 2.5 ms^{-1} is 0.5 s — N15/41/Q2

Answer: Distance travelled along the ground before coming to rest = 7.5 m

50



A small bead Q can move freely along a smooth horizontal straight wire AB of length 3 m. Three horizontal forces of magnitudes F N, 10 N and 20 N act on the bead in the directions shown in the diagram. The magnitude of the resultant of the three forces is R N in the direction shown in the diagram.

(i) Find the values of F and R. [5]

(ii) Initially the bead is at rest at A. It reaches B with a speed of 11.7 m s^{-1}. Find the mass of the bead. [3]

Answers: F = 1.90 and R = 12.4 *Answer*: Mass of the bead = 0.526 kg — N15/41/Q5

LAWS OF MOTION ON CONNECTED PARTICLES

EXAMPLE 6

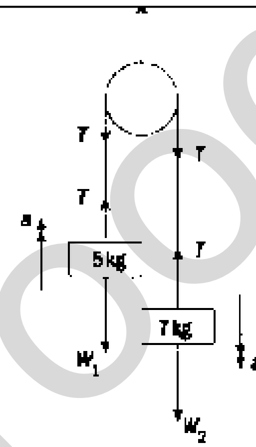
Two particles of mass 5 kg and 7 kg are connected by a light, inextensible string, which hangs over a smooth, light pulley.

Taking the value of g as $10\ \text{m s}^{-2}$, calculate

a) the weights of the two particles, W_1 and W_2

b) the acceleration of the system, a

c) the normal reaction at the point where the pulley is suspended, R.



EXAMPLE 7

Fig. 24.15 shows two particles A of mass I kg and B 2 kg connected by a light inelastic string which passes over a smooth pulley at C. The system is held at rest with B hanging freely while A is on a rough plane inclined at θ to the horizontal where $\tan\theta = \frac{3}{4}$. The coefficient of friction between A and the plane is 0.2. Find the magnitude of the acceleration of each particle and the tension in the string when the System is released.

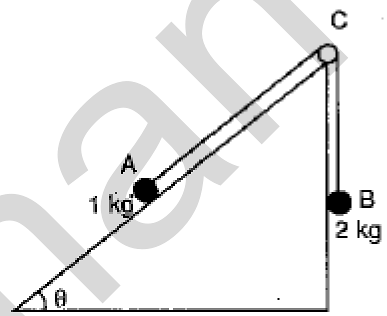


Fig. 24.15

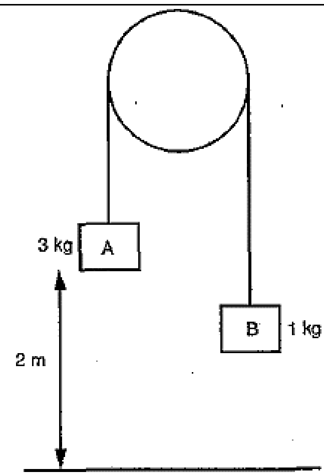
EXAMPLE 8

Particles A and B of masses 3 kg and 1 kg respectively (Fig. 24.18(a)) hang at the ends of a light string passing over a smooth pulley. A is released from rest when it is 2 m above the ground. Find

a) The common acceleration of the particles,

b) The speed of B when A reaches the ground,

c) How much higher B will travel afterwards. (It is assumed that B never reaches the pulley.)



EXAMPLE 9

A car of mass 1000 kg is pulling a trailer of mass 800 kg down a line of greatest slope of a hill. The hill is inclined at an angle of θ to the horizontal, where $\sin\theta = 0.05$. The car and the trailer are connected by a light, rigid tow-bar, which is parallel to the road. The driving force of the car's engine is 2000 N, the resistance to the car is 400 N, and the resistance to the trailer is 100 N. Find

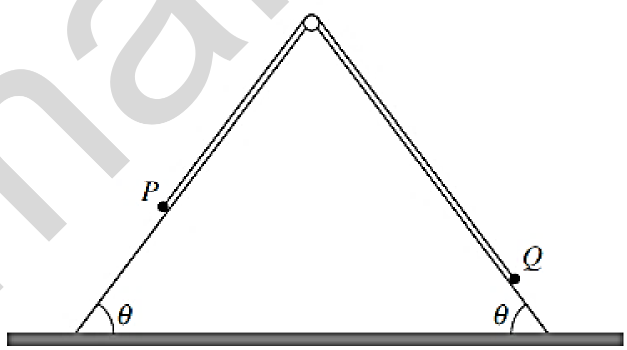
a) the acceleration of the system

b) the tension in the tow-bar.

(Take g as 10 m s^{-2}.)

EXAMPLE 10

J12/41/Q6



Particles P and Q, of masses 0.6 kg and 0.4 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a vertical cross-section of a triangular prism. The base of the prism is fixed on horizontal ground and each of the sloping sides is smooth. Each sloping side makes an angle θ with the ground, where $\sin\theta = 0.8$. Initially the particles are held at rest on the sloping sides, with the string taut (see diagram). The particles are released and move along lines of greatest slope.

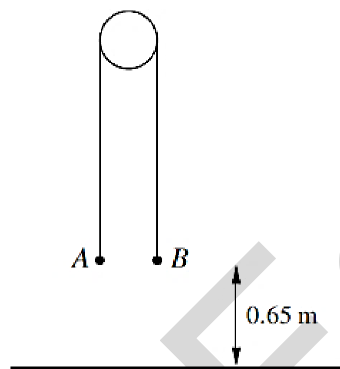
(i) Find the tension in the string and the acceleration of the particles while both are moving. [5]

The speed of P when it reaches the ground is $2\,\text{m s}^{-1}$. On reaching the ground P comes to rest and remains at rest. Q continues to move up the slope but does not reach the pulley.

(ii) Find the time taken from the instant that the particles are released until Q reaches its greatest height above the ground. [4]

<u>EXAMPLE 11</u>

J12/43/Q7



Two particles A and B have masses 0.12 kg and 0.38 kg respectively. The particles are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. A is held at rest with the string taut and both straight parts of the string vertical. A and B are each at a height of 0.65 m above horizontal ground (see diagram). A is released and B moves downwards. Find

(i) the acceleration of B while it is moving downwards, [2]

(ii) the speed with which B reaches the ground and the time taken for it to reach the ground. [3]

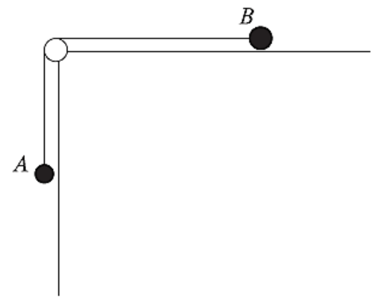
B remains on the ground while A continues to move with the string slack, without reaching the pulley. The string remains slack until A is at a height of 1.3 m above the ground for a second time. At this instant A has been in motion for a total time of T s.

(iii) Find the value of T and sketch the velocity-time graph for A for the first T s of its motion. [3]

(iv) Find the total distance travelled by A in the first T s of its motion. [2]

HOMEWORK: LAWS OF MOTION APPLIED TO PULLEYS
VARIANT 42

1



Particles A and B, of masses 0.2 kg and 0.3 kg respectively, are connected by a light inextensible string. The string passes over a smooth pulley at the edge of a rough horizontal table. Particle A hangs freely and particle B is in contact with the table (see diagram).

(i) The system is in limiting equilibrium with the string taut and A about to move downwards. Find the coefficient of friction between B and the table. [4]

A force now acts on particle B. This force has a vertical component of 1.8 N upwards and a horizontal component of X N directed away from the pulley.

(ii) The system is now in limiting equilibrium with the string taut and A about to move **upwards**. Find X. [3]

Answers: (i) $\frac{2}{3}$; (ii) 2.8.

J05/Q4

2

Particles P and Q are attached to opposite ends of a light inextensible string. P is at rest on a rough horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Q hangs vertically below the pulley (see diagram). The force exerted on the string by the pulley has magnitude $4\sqrt{2}$ N. The coefficient of friction between P and the table is 0.8.

(i) Show that the tension in the string is 4 N and state the mass of Q. [2]

(ii) Given that P is on the point of slipping, find its mass. [2]

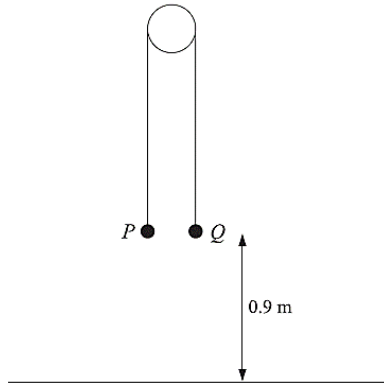
A particle of mass 0.1 kg is now attached to Q and the system starts to move.

(iii) Find the tension in the string while the particles are in motion. [4]

Answers: (i) 0.4 kg; (ii) 0.5 kg; (iii) 4.5 N.

J06/Q5

3



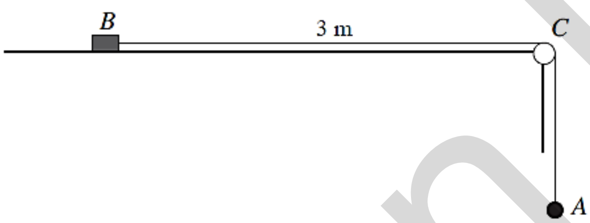
Particles P and Q, of masses 0.6 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed peg. The particles are held at rest with the string taut. Both particles are at a height of 0.9 m above the ground (see diagram). The system is released and each of the particles moves vertically. Find

(i) the acceleration of P and the tension in the string before P reaches the ground, [5]

(ii) the time taken for P to reach the ground. [2]

Answers: (i) 5 ms^{-2}; (ii) 0.6 s. J07/Q4

4



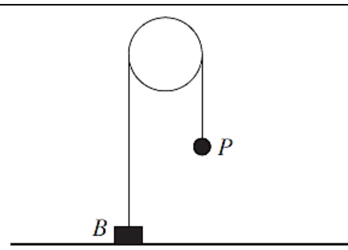
A block B of mass 0.6 kg and a particle A of mass 0.4 kg are attached to opposite ends of a light inextensible string. The block is held at rest on a rough horizontal table, and the coefficient of friction between the block and the table is 0.5. The string passes over a small smooth pulley C at the edge of the table and A hangs in equilibrium vertically below C. The part of the string between B and C is horizontal and the distance BC is 3 m (see diagram). B is released and the system starts to move.

(i) Find the acceleration of B and the tension in the string. [6]

(ii) Find the time taken for B to reach the pulley. [2]

Answers: (i) 1 ms^{-2}, 3.6 N; (ii) 2.45 s. J08/Q5

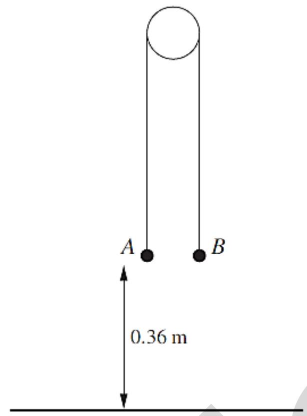
5



A block B of mass 5 kg is attached to one end of a light inextensible string. A particle P of mass 4 kg is attached to other end of the string. The string passes over a smooth pulley. The system is in equilibrium with the string taut and its straight parts vertical. B is at rest on the ground (see diagram). State the tension in the string and find the force exerted on B by the ground. [3]

Answers: 40 N, 10 N. **J09/Q1**

6



Particles A and B are attached to the ends of a light inextensible string which passes over a smooth pulley. The system is held at rest with the string taut and its straight parts vertical. Both particles are at a height of 0.36 m above the floor (see diagram). The system is released and A begins to fall, reaching the floor after 0.6 s.

(i) Find the acceleration of A as it falls. [2]

The mass of A is 0.45 kg. Find

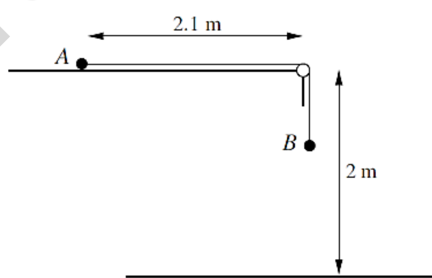
(ii) the tension in the string while A is falling, [2]

(iii) the mass of B, [3]

(iv) the maximum height above the floor reached by B. [3]

Answers: (i) 2 ms^{-2}; (ii) 3.6 N; (iii) 0.3 kg; (iv) 0.792 m. **J09/Q6**

7



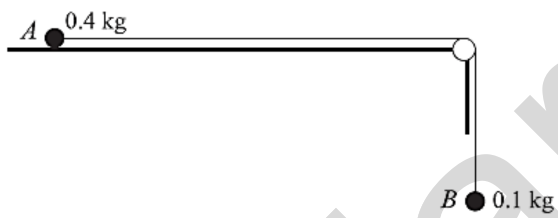
Particles A and B, of masses 0.2 kg and 0.45 kg respectively, are connected by a light inextensible string of length 2.8 m. The string passes over a small smooth pulley at the edge of a rough horizontal surface, which is 2 m above the floor. Particle A is held in contact with the surface at a distance of 2.1 m from the pulley and particle B hangs freely (see diagram). The coefficient of friction between A and the surface is 0.3. Particle A is released and the system begins to move.

(i) Find the acceleration of the particles and show that the speed of B immediately before it hits the floor is $3.95\ \text{m s}^{-1}$, correct to 3 significant figures. [7]

(ii) Given that B remains on the floor, find the speed with which A reaches the pulley. [4]

Answers: (i) $6\ \text{ms}^{-2}$; (ii) $3.29\ \text{ms}^{-1}$. **J10/42/Q6**

8



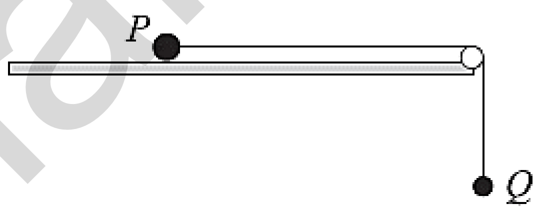
Particles A and B, of masses 0.4 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest on a horizontal table with the string passing over a smooth pulley at the edge of the table. Particle B hangs vertically below the pulley (see diagram). The system is released from rest. In the subsequent motion a constant frictional force of magnitude 0.6 N acts on A. Find

(i) the tension in the string, [4]

(ii) the speed of B 1.5 s after it starts to move. [3]

Answers: (i) 0.92 N; (ii) $1.2\ \text{ms}^{-1}$. **N03/Q5**

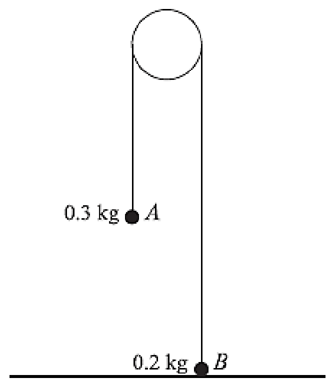
9



Two particles P and Q, of masses 1.7 kg and 0.3 kg respectively, are connected by a light inextensible string. P is held on a smooth horizontal table with the string taut and passing over a small smooth pulley fixed at the edge of the table. Q is at rest vertically below the pulley. P is released. Find the acceleration of the particles and the tension in the string. [5]

Answer: $1.5\ \text{ms}^{-2}$, 2.55 N. **N04/Q1**

10



Two particles A and B, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. Particle B is held on the horizontal floor and particle A hangs in equilibrium. Particle B is released and each particle starts to move vertically with constant acceleration of magnitude $a\,\mathrm{m\,s^{-2}}$.

(i) Find the value of a. [4]

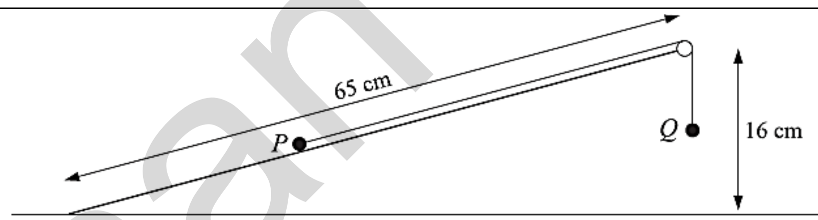
Particle A hits the floor 1.2 s after it starts to move, and does not rebound upwards.

(ii) Show that A hits the floor with a speed of $2.4\,\mathrm{m\,s^{-1}}$. [1]

(iii) Find the gain in gravitational potential energy by B, from leaving the floor until reaching its greatest height. [5]

N05/Q7

11



A rough inclined plane of length 65 cm is fixed with one end at a height of 16 cm above the other end. Particles P and Q, of masses 0.13 kg and 0.11 kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth pulley at the top of the plane. Particle P is held at rest on the plane and particle Q hangs vertically below the pulley (see diagram). The system is released from rest and P starts to move up the plane.

(i) Draw a diagram showing the forces acting on P during its motion up the plane. [1]

(ii) Show that $T - F > 0.32$, where T N is the tension in the string and F N is the magnitude of the frictional force on P. [4]

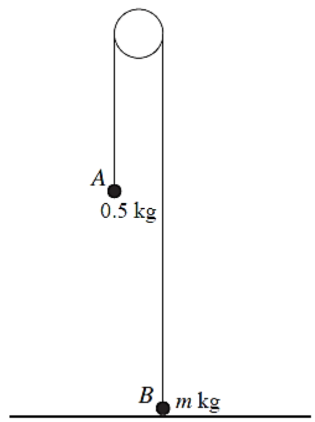
The coefficient of friction between P and the plane is 0.6.

(iii) Find the acceleration of P. [6]

Answer. (iii) $0.1\,\mathrm{ms^{-2}}$.

N07/Q7

12



Particles A and B, of masses 0.5 kg and m kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. Particle B is held at rest on the horizontal floor and particle A hangs in equilibrium (see diagram). B is released and each particle starts to move vertically. A hits the floor 2 s after B is released. The speed of each particle when A hits the floor is $5\,\text{m s}^{-1}$.

(i) For the motion while A is moving downwards, find

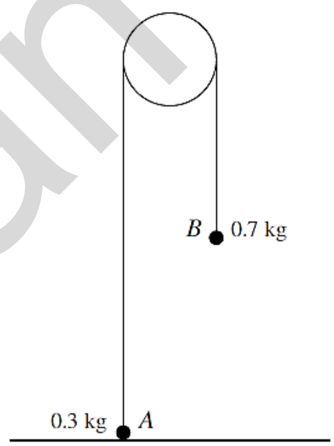
(a) the acceleration of A, [2]

(b) the tension in the string. [3]

(ii) Find the value of m. [3]

Answers: (i)(a) 2.5 ms^{-2}, (b) 3.75 N; (ii) 0.3. **N08/Q5**

13



Particles **A** and **B**, of masses 0.3 kg and 0.7 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. Particle **A** is held on the horizontal floor and particle **B** hangs in equilibrium. Particle **A** is released and both particles start to move vertically.

(i) Find the acceleration of the particles. [3]

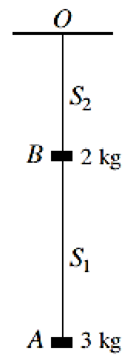
The speed of the particles immediately before **B** hits the floor is $1.6\,\text{m s}^{-1}$. Given that **B** does not rebound upwards, find

(ii) the maximum height above the floor reached by **A**, [3]

(iii) the time taken by **A**, from leaving the floor, to reach this maximum height. [3]

Answers: **(i)** 4 ms^{-2}; **(ii)** 0.448 m; **(iii)** 0.56 s. **N09/42/Q6**

14



A block A of mass 3 kg is attached to one end of a light inextensible string S_1. Another block B of mass 2 kg is attached to the other end of S_1, and is also attached to one end of another light inextensible string S_2. The other end of S_2 is attached to a fixed point O and the blocks hang in equilibrium below O (see diagram).

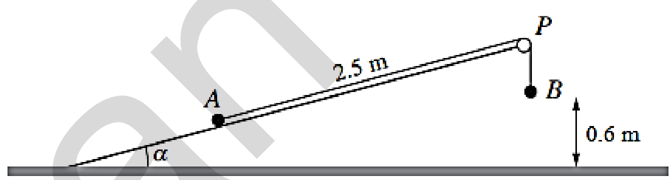
(i) Find the tension in S_1 and the tension in S_2. [2]

The string S_2 breaks and the particles fall. The air resistance on A is 1.6 N and the air resistance on B is 4 N.

(ii) Find the acceleration of the particles and the tension in S_1. [5]

Answers: **(i)** S_1 = 30 N, S_2 = 50 N **(ii)** a = 8.88 ms^{-2}, T = 1.76 N **J12/42/Q5**

15



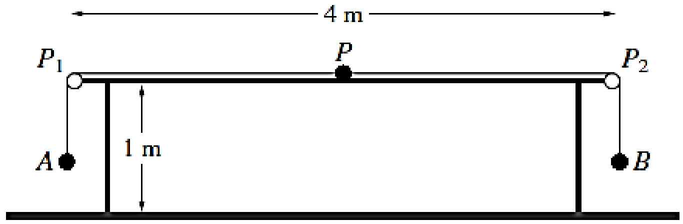
Particles A of mass 0.26 kg and B of mass 0.52 kg are attached to the ends of a light inextensible string. The string passes over a small smooth pulley P which is fixed at the top of a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{16}{65}$ and $\cos\alpha = \frac{63}{65}$. A is held at rest at a point 2.5 metres from P, with the part AP of the string parallel to a line of greatest slope of the plane. B hangs freely below P at a point 0.6 m above the floor (see diagram). A is released and the particles start to move. Find

(i) the magnitude of the acceleration of the particles and the tension in the string, [5]

(ii) the speed with which B reaches the floor and the distance of A from P when A comes to instantaneous rest. [6]

Answers: **(i)** Acceleration = 5.85 ms^{-2}, Tension in string = 2.16N **J13/42/Q7**
(ii) Speed of B as it reaches the floor = 2.65 ms^{-1}, Distance of A from P = 0.475m

16



A light inextensible string of length 5.28 m has particles A and B, of masses 0.25 kg and 0.75 kg respectively, attached to its ends. Another particle P, of mass 0.5 kg, is attached to the mid-point of the string. Two small smooth pulleys P_1 and P_2 are fixed at opposite ends of a rough horizontal table of length 4 m and height 1 m. The string passes over P_1 and P_2 with particle A held at rest vertically below P_1, the string taut and B hanging freely below P_2. Particle P is in contact with the table halfway between P_1 and P_2 (see diagram). The coefficient of friction between P and the table is 0.4. Particle A is released and the system starts to move with constant acceleration of magnitude a m s^{-2}. The tension in the part AP of the string is T_A N and the tension in the part PB of the string is T_B N.

(i) Find T_A and T_B in terms of a. [3]

(ii) Show by considering the motion of P that $a = 2$. [3]

(iii) Find the speed of the particles immediately before B reaches the floor. [2]

(iv) Find the deceleration of P immediately after B reaches the floor. [2]

Answer: $T_A = 2.5 + 0.25a$, $T_B = 7.5 - 0.75a$ *Answer:* given **J14/42/Q7**

Answer: Speed = 1.2 ms^{-1} *Answer:* Deceleration = 6 ms^{-2}

17

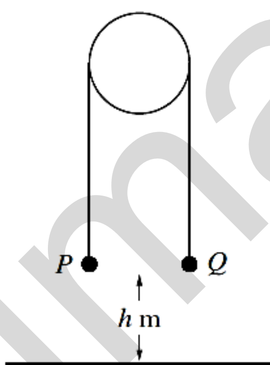


Fig. 1

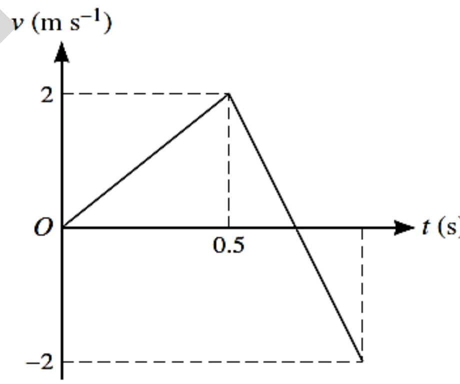


Fig. 2

Two particles P and Q have masses m kg and $(1 - m)$ kg respectively. The particles are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. P is held at rest with the string taut and both straight parts of the string vertical. P and Q are each at a height of h m above horizontal ground (see Fig. 1). P is released and Q moves downwards. Subsequently Q hits the ground and comes to rest. Fig. 2 shows the velocity-time graph for P while Q is moving downwards or is at rest on the ground.

(i) Find the value of h. [2]

(ii) Find the value of m, and find also the tension in the string while Q is moving. [6]

(iii) The string is slack while Q is at rest on the ground. Find the total time from the instant that P is released until the string becomes taut again. [3]

Answer: **(i)** h = 0.5 m **(ii)** m = 0.3 Tension = 4.2 N **(iii)** Total time taken = 0.9 seconds **J15/42/Q6**

18 Particles A and B, of masses 0.9 kg and 0.6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley. The system is released from rest with the string taut, with its straight parts vertical and with the particles at the same height above the horizontal floor. In the subsequent motion, B does not reach the pulley.

(i) Find the acceleration of A and the tension in the string during the motion before A hits the floor. [4]

After A hits the floor, B continues to move vertically upwards for a further 0.3 s.

(ii) Find the height of the particles above the floor at the instant that they started to move. [4]

Answers: **(i)** 2 ms^{-2}, 7.2 N; **(ii)** 2.25 m **N11/42/Q5**

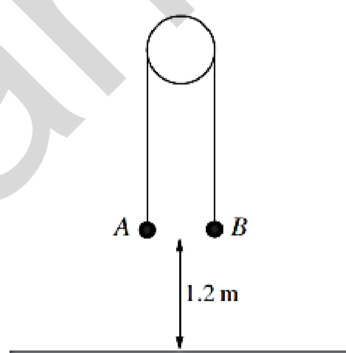
19 Particles A and B of masses m kg and $(1 - m)$ kg respectively are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. The system is released from rest with the straight parts of the string vertical. A moves vertically downwards and 0.3 seconds later it has speed 0.6 m s^{-1}. Find

(i) the acceleration of A, [2]

(ii) the value of m and the tension in the string. [4]

Answers: **(i)** 2 ms^{-2}; **(ii)** m = 0.6, tension = 4.8 N. **N12/42/Q2**

20



Particles A of mass 0.4 kg and B of mass 1.6 kg are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. A is held at rest and B hangs freely, with both straight parts of the string vertical and both particles at a height of 1.2 m above the floor (see diagram). A is released and both particles start to move.

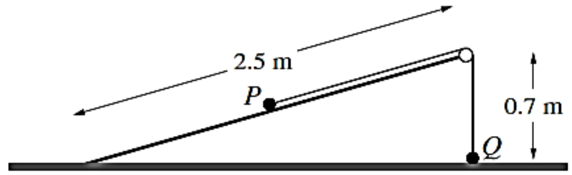
(i) Find the work done on B by the tension in the string, as B moves to the floor. [5]

When particle B reaches the floor it remains at rest. Particle A continues to move upwards.

(ii) Find the greatest height above the floor reached by particle A. [4]

Answer: Work done by tension = 7.68 J *Answer*: Greatest height above floor = 3.12 m N13/42/Q6

21



A smooth inclined plane of length 2.5 m is fixed with one end on the horizontal floor and the other end at a height of 0.7 m above the floor. Particles P and Q, of masses 0.5 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth pulley fixed at the top of the plane. Particle Q is held at rest on the floor vertically below the pulley. The string is taut and P is at rest on the plane (see diagram). Q is released and starts to move vertically upwards towards the pulley and P moves down the plane.

(i) Find the tension in the string and the magnitude of the acceleration of the particles before Q reaches the pulley. [5]

At the instant just before Q reaches the pulley the string breaks; P continues to move down the plane and reaches the floor with a speed of $2\,\text{m}\,\text{s}^{-1}$.

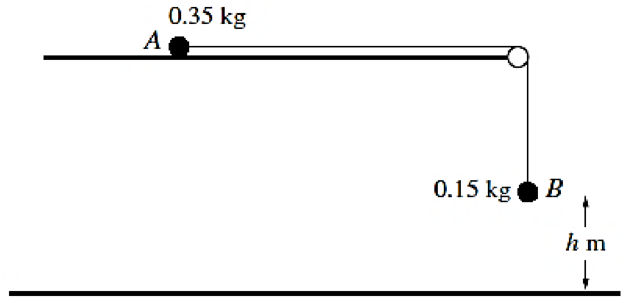
(ii) Find the length of the string. [3]

Answer: The tension in the string $T = \frac{16}{15} = 1.07$ N Magnitude of acceleration $a = \frac{2}{3} = 0.667\ \text{ms}^{-2}$ N15/42/Q5

Answer: The length of the string is 1.95 m

HOMEWORK: LAWS OF MOTION APPLIED TO PULLEYS
VARIANT 41 & 43

1



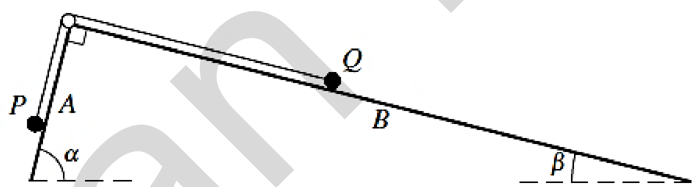
Particles A and B, of masses 0.35 kg and 0.15 kg respectively, are attached to the ends of a light inextensible string. A is held at rest on a smooth horizontal surface with the string passing over a small smooth pulley fixed at the edge of the surface. B hangs vertically below the pulley at a distance h m above the floor (see diagram). A is released and the particles move. B reaches the floor and A subsequently reaches the pulley with a speed of $3\,\text{m s}^{-1}$.

(i) Explain briefly why the speed with which B reaches the floor is $3\,\text{m s}^{-1}$. [1]

(ii) Find the value of h. [4]

Answer: **(ii)** h = 1.5 43/J15/2

2



The tops of each of two smooth inclined planes A and B meet at a right angle. Plane A is inclined at angle α to the horizontal and plane B is inclined at angle β to the horizontal, where $\sin\alpha = \frac{63}{65}$ and $\sin\beta = \frac{16}{65}$. A small smooth pulley is fixed at the top of the planes and a light inextensible string passes over the pulley. Two particles P and Q, each of mass 0.65 kg, are attached to the string, one at each end. Particle Q is held at rest at a point of the same line of greatest slope of the plane B as the pulley. Particle P rests freely below the pulley in contact with plane A (see diagram). Particle Q is released and the particles start to move with the string taut. Find the tension in the string. [5]

Answer: 3.95 43/N14/2

3

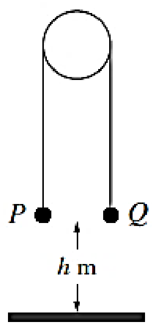


Fig. 1

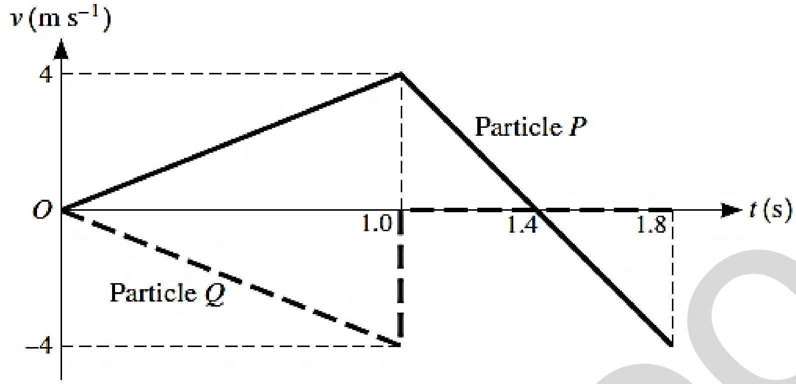


Fig. 2

Particles P and Q have a total mass of 1 kg. The particles are attached to opposite ends of a light inextensible string which passes over a smooth fixed pulley. P is held at rest and Q hangs freely, with both straight parts of the string vertical. Both particles are at a height of h m above the floor (see Fig. 1). P is released from rest and the particles start to move with the string taut. Fig. 2 shows the velocity-time graphs for P's motion and for Q's motion, where the positive direction for velocity is vertically upwards. Find

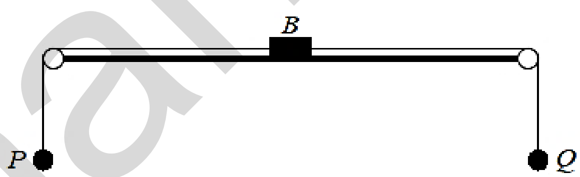
(i) the magnitude of the acceleration with which the particles start to move and the mass of each of the particles, [5]

(ii) the value of h, [1]

(iii) the greatest height above the floor reached by particle P. [2]

Answer: 4, 0.3, 0.7 43/N14/6

4



A small block B of mass 0.25 kg is attached to the mid-point of a light inextensible string. Particles P and Q, of masses 0.2 kg and 0.3 kg respectively, are attached to the ends of the string. The string passes over two smooth pulleys fixed at opposite sides of a rough table, with B resting in limiting equilibrium on the table between the pulleys and particles P and Q and block B are in the same vertical plane (see diagram).

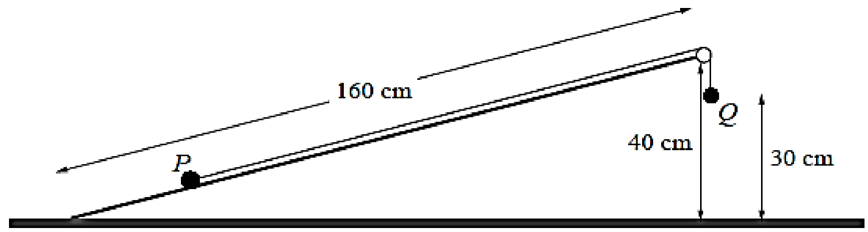
(i) Find the coefficient of friction between B and the table. [3]

Q is now removed so that P and B begin to move.

(ii) Find the acceleration of P and the tension in the part PB of the string. [6]

Answer: 0.4, 2.22, 1.56 41/N14/5

5



A smooth inclined plane of length 160 cm is fixed with one end at a height of 40 cm above the other end, which is on horizontal ground. Particles P and Q, of masses 0.76 kg and 0.49 kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth pulley fixed at the top of the plane. Particle P is held at rest on the same line of greatest slope as the pulley and Q hangs vertically below the pulley at a height of 30 cm above the ground (see diagram). P is released from rest. It starts to move up the plane and does not reach the pulley. Find

(i) the acceleration of the particles and the tension in the string before Q reaches the ground, [4]

(ii) the speed with which Q reaches the ground, [2]

(iii) the total distance travelled by P before it comes to instantaneous rest. [3]

Answer: 2.4 ms^{-2} 3.72 N 1.2 ms^{-1} 0.588 m 43/J14/7

6

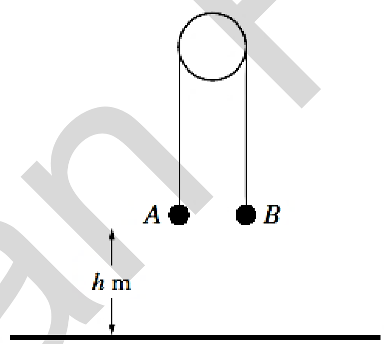


Fig. 1

Particles A of mass 0.25 kg and B of mass 0.75 kg are attached to opposite ends of a light inextensible string which passes over a fixed smooth pulley. The system is held at rest with the string taut and its straight parts vertical. Both particles are at a height of h m above the floor (see Fig. 1). The system is released from rest, and 0.6 s later, when both particles are in motion, the string breaks. The particle A does not reach the pulley in the subsequent motion.

(i) Find the acceleration of A and the distance travelled by A before the string breaks. [4]

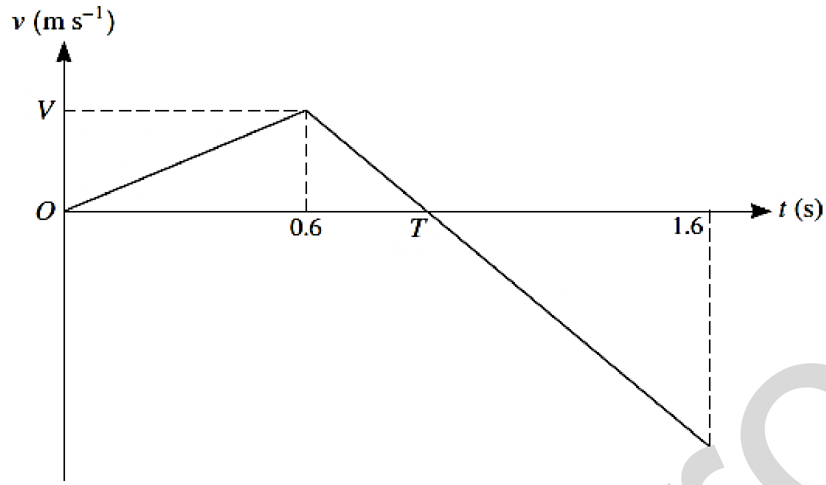


Fig. 2

The velocity-time graph shown in Fig. 2 is for the motion of particle A until it hits the floor. The velocity of A when the string breaks is $V\,\mathrm{m\,s^{-1}}$ and T s is the time taken for A to reach its greatest height.

(ii) Find the value of V and the value of T. [3]

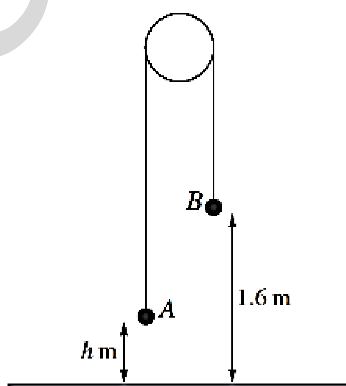
(iii) Find the distance travelled by A upwards and the distance travelled by A downwards and hence find h. [3]

Answer: Acceleration = 5 ms^{-2}. Distance travelled = 0.9 m 41/J14/6

Answer: V = 3, T = 0.9

Answer: Distance travelled upwards = 1.35 m, distance travelled downwards = 2.45 m, h = 2.45 – 1.35 = 1.1

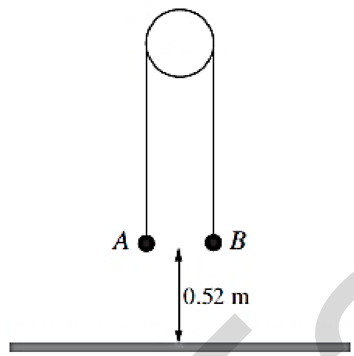
7



Particle A of mass 0.2 kg and particle B of mass 0.6 kg are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley. B is held at rest at a height of 1.6 m above the floor. A hangs freely at a height of h m above the floor. Both straight parts of the string are vertical (see diagram). B is released and both particles start to move. When B reaches the floor it remains at rest, but A continues to move vertically upwards until it reaches a height of 3 m above the floor. Find the speed of B immediately before it hits the floor, and hence find the value of h. [6]

Answer. 4 ms^{-1} 0.6 43/N13/2

8



Particles A and B, of masses 0.3 kg and 0.7 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley. A is held at rest and B hangs freely, with both straight parts of the string vertical and both particles at a height of 0.52 m above the floor (see diagram). A is released and both particles start to move.

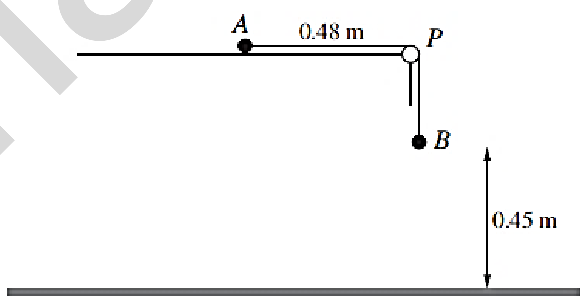
(i) Find the tension in the string. [4]

When both particles are moving with speed 1.6 m s^{-1} the string breaks.

(ii) Find the time taken, from the instant that the string breaks, for A to reach the floor. [5]

Answer: (i) 4.2 N (ii) 0.6 s 41/N13/6

9



Particle A of mass 1.26 kg and particle B of mass 0.9 kg are attached to the ends of a light inextensible string. The string passes over a small smooth pulley P which is fixed at the edge of a rough horizontal table. A is held at rest at a point 0.48 m from P, and B hangs vertically below P, at a height of 0.45 m above the floor (see diagram). The coefficient of friction between A and the table is $\frac{2}{7}$. A is released and the particles start to move.

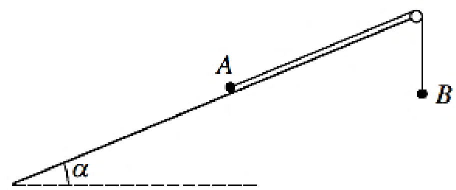
(i) Show that the magnitude of the acceleration of the particles is 2.5 m s^{-2} and find the tension in the string. [5]

(ii) Find the speed with which B reaches the floor. [2]

(iii) Find the speed with which A reaches the pulley. [4]

Answer: 2.5 ms^{-2}; 6.75 N; 1.5 ms^{-1}; 1.44 ms^{-1} 43/J13/7

10



A light inextensible string has a particle A of mass 0.26 kg attached to one end and a particle B of mass 0.54 kg attached to the other end. The particle A is held at rest on a rough plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{5}{13}$. The string is taut and parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley at the top of the plane. Particle B hangs at rest vertically below the pulley (see diagram). The coefficient of friction between A and the plane is 0.2. Particle A is released and the particles start to move.

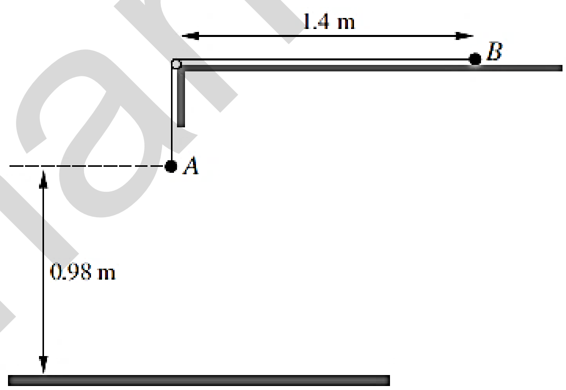
(i) Find the magnitude of the acceleration of the particles and the tension in the string. [6]

Particle A reaches the pulley 0.4 s after starting to move.

(ii) Find the distance moved by each of the particles. [2]

Answer: i) 2.75 ii) 0.392 41/J13/5

11



Particles A and B have masses 0.32 kg and 0.48 kg respectively. The particles are attached to the ends of a light inextensible string which passes over a small smooth pulley fixed at the edge of a smooth horizontal table. Particle B is held at rest on the table at a distance of 1.4 m from the pulley. A hangs vertically below the pulley at a height of 0.98 m above the floor (see diagram). A, B, the string and the pulley are all in the same vertical plane. B is released and A moves downwards.

(i) Find the acceleration of A and the tension in the string. [5]

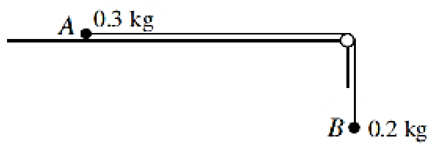
A hits the floor and B continues to move towards the pulley. Find the time taken, from the instant that B is released, for

(ii) A to reach the floor, [2]

(iii) B to reach the pulley. [3]

Answers: (i) $a = 4\ ms^{-2}$, $t = 1.92$ N; (ii) 0.7 s; (iii) 0.85 s. 43/N12/7

12



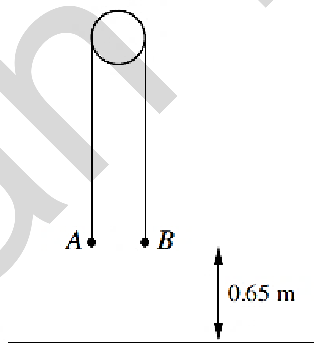
Particles A and B, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. A is held at rest on a rough horizontal table with the string passing over a small smooth pulley at the edge of the table. B hangs vertically below the pulley (see diagram). The system is released and B starts to move downwards with acceleration $1.6\ m\,s^{-2}$. Find

(i) the tension in the string after the system is released, [2]

(ii) the frictional force acting on A. [3]

Answers: (i) 1.68 N; (ii) 1.2 N. 41/N12/2

13



Two particles A and B have masses 0.12 kg and 0.38 kg respectively. The particles are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. A is held at rest with the string taut and both straight parts of the string vertical. A and B are each at a height of 0.65 m above horizontal ground (see diagram). A is released and B moves downwards. Find

(i) the acceleration of B while it is moving downwards, [2]

(ii) the speed with which B reaches the ground and the time taken for it to reach the ground. [3]

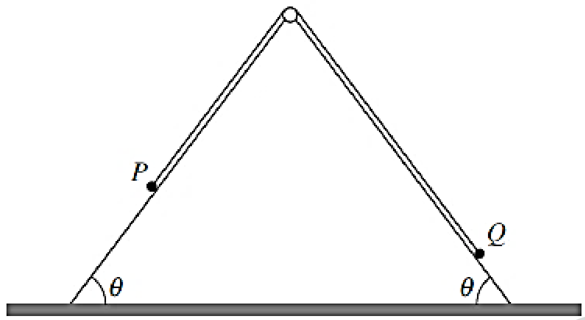
B remains on the ground while A continues to move with the string slack, without reaching the pulley. The string remains slack until A is at a height of 1.3 m above the ground for a second time. At this instant A has been in motion for a total time of T s.

(iii) Find the value of T and sketch the velocity-time graph for A for the first T s of its motion. [3]

(iv) Find the total distance travelled by A in the first T s of its motion. [2]

Answers: **(i)** 5.2 ms^{-2} **(ii)** 2.6 ms^{-1} 0.5 s **(iii)** 1.02 **(iv)** 1.326 m (1.33 m) 43/J12/7

14



Particles P and Q, of masses 0.6 kg and 0.4 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a vertical cross-section of a triangular prism. The base of the prism is fixed on horizontal ground and each of the sloping sides is smooth. Each sloping side makes an angle θ with the ground, where $\sin\theta = 0.8$. Initially the particles are held at rest on the sloping sides, with the string taut (see diagram). The particles are released and move along lines of greatest slope.

(i) Find the tension in the string and the acceleration of the particles while both are moving. [5]

The speed of P when it reaches the ground is $2\,\text{m s}^{-1}$. On reaching the ground P comes to rest and remains at rest. Q continues to move up the slope but does not reach the pulley.

(ii) Find the time taken from the instant that the particles are released until Q reaches its greatest height above the ground. [4]

Answers: **(i)** Tension = 3.84 N, acceleration = 1.6 ms^{-2} **(ii)** 1.5 s 41/J12/6

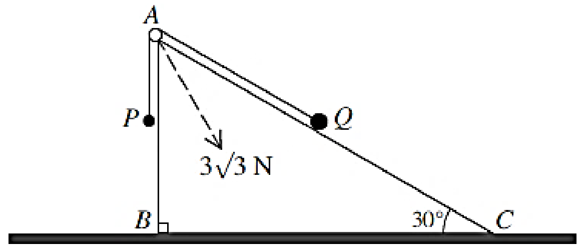
15 Particles P and Q are attached to opposite ends of a light inextensible string which passes over a fixed smooth pulley. The system is released from rest with the string taut, with its straight parts vertical, and with both particles at a height of 2 m above horizontal ground. P moves vertically downwards and does not rebound when it hits the ground. At the instant that P hits the ground, Q is at the point X, from where it continues to move vertically upwards without reaching the pulley. Given that P has mass 0.9 kg and that the tension in the string is 7.2 N while P is moving, find the total distance travelled by Q from the instant it first reaches X until it returns to X. [6]

Answer: 0.8 m 43/N11/3

16 Particles A of mass 0.65 kg and B of mass 0.35 kg are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. B is held at rest with the string taut and both of its straight parts vertical. The system is released from rest and the particles move vertically. Find the tension in the string and the magnitude of the resultant force exerted on the pulley by the string. [5]

Answers: Tension = 4.55 N, Magnitude = 9.1 N 41/N11/2

17



A small smooth pulley is fixed at the highest point A of a cross-section ABC of a triangular prism. Angle $ABC = 90°$ and angle $BCA = 30°$. The prism is fixed with the face containing BC in contact with a horizontal surface. Particles P and Q are attached to opposite ends of a light inextensible string, which passes over the pulley. The particles are in equilibrium with P hanging vertically below the pulley and Q in contact with AC. The resultant force exerted on the pulley by the string is $3\sqrt{3}$ N (see diagram).

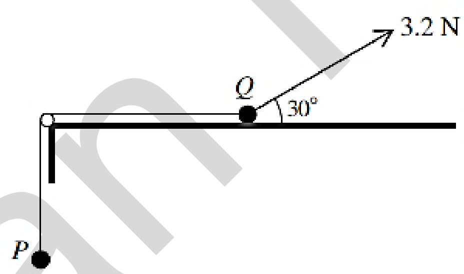
(i) Show that the tension in the string is 3 N. [2]

The coefficient of friction between Q and the prism is 0.75.

(ii) Given that Q is in limiting equilibrium and on the point of moving upwards, find its mass. [5]

Answer. (ii) 0.261 kg. 43/N10/3

18



Particles P and Q, of masses 0.2 kg and 0.5 kg respectively, are connected by a light inextensible string. The string passes over a smooth pulley at the edge of a rough horizontal table. P hangs freely and Q is in contact with the table. A force of magnitude 3.2 N acts on Q, upwards and away from the pulley, at an angle of 30° to the horizontal (see diagram).

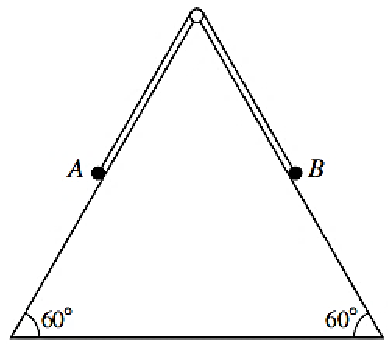
(i) The system is in limiting equilibrium with P about to move upwards. Find the coefficient of friction between Q and the table. [6]

The force of magnitude 3.2 N is now removed and P starts to move downwards.

(ii) Find the acceleration of the particles and the tension in the string. [4]

Answer: i) 0.227 ii) 1.75 41/N10/7

19



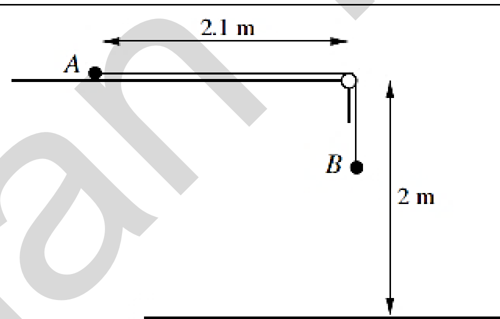
The diagram shows a vertical cross-section of a triangular prism which is fixed so that two of its faces are inclined at 60° to the horizontal. One of these faces is smooth and one is rough. Particles A and B, of masses 0.36 kg and 0.24 kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth pulley fixed at the highest point of the cross-section. B is held at rest at a point of the cross-section on the rough face and A hangs freely in contact with the smooth face (see diagram). B is released and starts to move up the face with acceleration $0.25\,\text{m s}^{-2}$.

(i) By considering the motion of A, show that the tension in the string is 3.03 N, correct to 3 significant figures. [2]

(ii) Find the coefficient of friction between B and the rough face, correct to 2 significant figures. [6]

Answer: (ii) 0.74. 43/J10/4

20



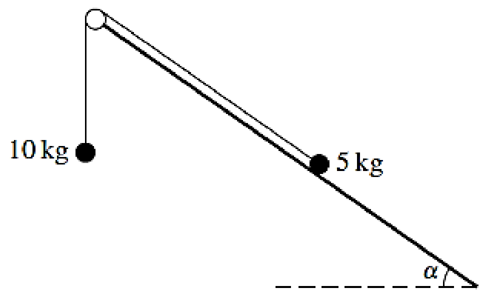
Particles A and B, of masses 0.2 kg and 0.45 kg respectively, are connected by a light inextensible string of length 2.8 m. The string passes over a small smooth pulley at the edge of a rough horizontal surface, which is 2 m above the floor. Particle A is held in contact with the surface at a distance of 2.1 m from the pulley and particle B hangs freely (see diagram). The coefficient of friction between A and the surface is 0.3. Particle A is released and the system begins to move.

(i) Find the acceleration of the particles and show that the speed of B immediately before it hits the floor is $3.95\,\text{m s}^{-1}$, correct to 3 significant figures. [7]

(ii) Given that B remains on the floor, find the speed with which A reaches the pulley. [4]

Answer: i) 6 ii) 3.29 41/J10/6

21



Two particles of masses 5 kg and 10 kg are connected by a light inextensible string that passes over a fixed smooth pulley. The 5 kg particle is on a rough fixed slope which is at an angle of α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The 10 kg particle hangs below the pulley (see diagram). The coefficient of friction between the slope and the 5 kg particle is $\frac{1}{2}$. The particles are released from rest. Find the acceleration of the particles and the tension in the string. [7]

Answers: $a = \frac{10}{3} = 3.33 \text{ ms}^{-2}$ $T = \frac{200}{3} = 66.7$ N J16/41/Q5

---

22 Two particles of masses 1.3 kg and 0.7 kg are connected by a light inextensible string that passes over a fixed smooth pulley. The particles are held at the same vertical height with the string taut. The distance of each particle above a horizontal plane is 2 m, and the distance of each particle below the pulley is 4 m. The particles are released from rest.

(i) Find

(a) the tension in the string before the particle of mass 1.3 kg reaches the plane,

(b) the time taken for the particle of mass 1.3 kg to reach the plane.

[6]

(ii) Find the greatest height of the particle of mass 0.7 kg above the plane. [4]

Answer: 9.1 N 1.15 s 4.6 m J16/43/Q6

---

23 A particle P moves in a straight line. At time t s, the displacement of P from O is s m and the acceleration of P is a m s^{-2}, where $a = 6t - 2$. When $t = 1$, $s = 7$ and when $t = 3$, $s = 29$.

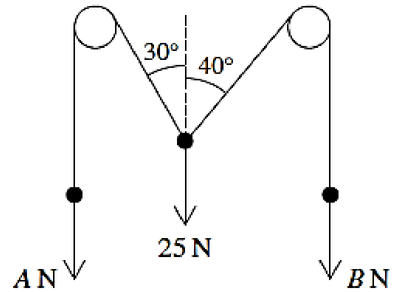
(i) Find the set of values of t for which the particle is decelerating. [2]

(ii) Find s in terms of t. [5]

(iii) Find the time when the velocity of the particle is 10 m s^{-1}. [3]

Answer: $t<1/3$ $s = t^3 - t^2 + 2t + 5$ 2s J16/43/Q7

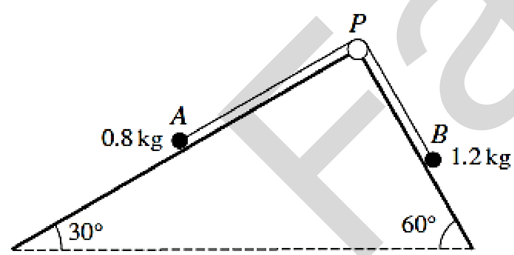
24



Two light inextensible strings are attached to a particle of weight 25 N. The strings pass over two smooth fixed pulleys and have particles of weights A N and B N hanging vertically at their ends. The sloping parts of the strings make angles of 30° and 40° respectively with the vertical (see diagram). The system is in equilibrium. Find the values of A and B. [6]

Answers: $A = 17.1$, $B = 13.3$ (to three significant figures) J17/41/Q3

25



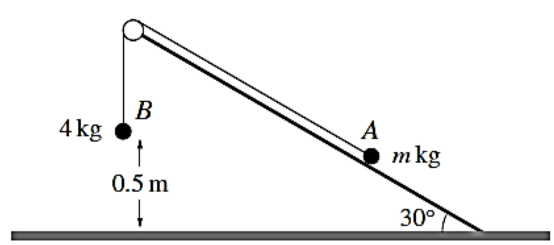
As shown in the diagram, a particle A of mass 0.8 kg lies on a plane inclined at an angle of 30° to the horizontal and a particle B of mass 1.2 kg lies on a plane inclined at an angle of 60° to the horizontal. The particles are connected by a light inextensible string which passes over a small smooth pulley P fixed at the top of the planes. The parts AP and BP of the string are parallel to lines of greatest slope of the respective planes. The particles are released from rest with both parts of the string taut.

(i) Given that both planes are smooth, find the acceleration of A and the tension in the string. [6]

(ii) It is given instead that both planes are rough, with the same coefficient of friction, μ, for both particles. Find the value of μ for which the system is in limiting equilibrium. [6]

Answers: **(i)** The acceleration of A is 3.20 ms^{-2} (to three significant figures) The tension in the string is 6.56 N (to three significant figures) **(ii)** The value of μ for which the system is in limiting equilibrium is 0.494 (to three significant figures) J17/41/Q7

26



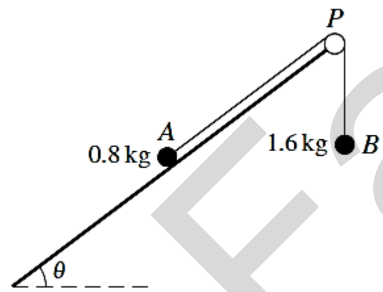
Two particles A and B of masses m kg and 4 kg respectively are connected by a light inextensible string that passes over a fixed smooth pulley. Particle A is on a rough fixed slope which is at an angle of 30° to the horizontal ground. Particle B hangs vertically below the pulley and is 0.5 m above the ground (see diagram). The coefficient of friction between the slope and particle A is 0.2.

(i) In the case where the system is in equilibrium with particle A on the point of moving directly up the slope, show that $m = 5.94$, correct to 3 significant figures. [6]

(ii) In the case where $m = 3$, the system is released from rest with the string taut. Find the total distance travelled by A before coming to instantaneous rest. You may assume that A does not reach the pulley. [8]

Answer: 0.710 m J17/43/Q7

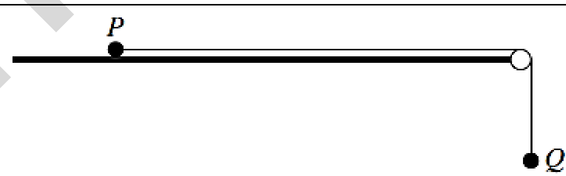
27



Two particles A and B, of masses 0.8 kg and 1.6 kg respectively, are connected by a light inextensible string. Particle A is placed on a smooth plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The string passes over a small smooth pulley P fixed at the top of the plane, and B hangs freely (see diagram). The section AP of the string is parallel to a line of greatest slope of the plane. The particles are released from rest with both sections of the string taut. Use an energy method to find the speed of the particles after each particle has moved a distance of 0.5 m, assuming that A has not yet reached the pulley. [6]

Answer: 2.16 ms^{-1} J18/43/Q4

28



Two particles P and Q, of masses 0.6 kg and 0.4 kg respectively, are connected by a light inextensible string. The string passes over a small smooth light pulley fixed at the edge of a smooth horizontal table. Initially P is held at rest on the table and Q hangs vertically (see diagram). P is then released. Find the tension in the string and the acceleration of Q. [4]

Answers: The tension in the string is 2.4 N The acceleration of Q is 4 ms^{-2} N16/41/Q1

29

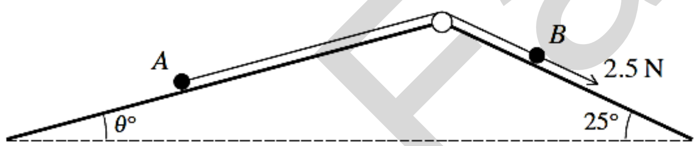
Particles P and Q, of masses 7 kg and 3 kg respectively, are attached to the two ends of a light inextensible string. The string passes over two small smooth pulleys attached to the two ends of a horizontal table. The two particles hang vertically below the two pulleys. The two particles are both initially at rest, 0.5 m below the level of the table, and 0.4 m above the horizontal floor (see diagram).

(i) Find the acceleration of the particles and the speed of P immediately before it reaches the floor. [4]

(ii) Determine whether Q comes to instantaneous rest before it reaches the pulley directly above it. [2]

Answer: Acceleration of the particles is 4 ms^{-2} Speed of P is 1.79 ms^{-1} N16/43/Q3

30



Two particles A and B of masses 0.9 kg and 0.4 kg respectively are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the top of two inclined planes. The particles are initially at rest with A on a smooth plane inclined at angle $\theta°$ to the horizontal and B on a plane inclined at angle 25° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes. A force of magnitude 2.5 N is applied to B acting down the plane (see diagram).

(i) For the case where $\theta = 15$ and the plane on which B rests is smooth, find the acceleration of B. [5]

(ii) For a different value of θ, the plane on which B rests is rough with coefficient of friction between the plane and B of 0.8. The system is in limiting equilibrium with B on the point of moving in the direction of the 2.5 N force. Find the value of θ. [5]

Answer: The acceleration of B is 1.43 m s^{-2} (to 3sf) *Answer:* $\theta = 8.2$ (to 1dp) N17/41/Q7

31 Two particles A and B have masses 0.35 kg and 0.45 kg respectively. The particles are attached to the ends of a light inextensible string which passes over a small fixed smooth pulley which is 1 m above horizontal ground. Initially particle A is held at rest on the ground vertically below the pulley, with the string taut. Particle B hangs vertically below the pulley at a height of 0.64 m above the ground. Particle A is released.

(i) Find the speed of A at the instant that B reaches the ground. [5]

(ii) Assuming that B does not bounce after it reaches the ground, find the total distance travelled by A between the instant that B reaches the ground and the instant when the string becomes taut again. [2]

Answers: (i) $1.26\,\text{ms}^{-1}$ (ii) $0.16\,\text{m}$ N17/43/Q4

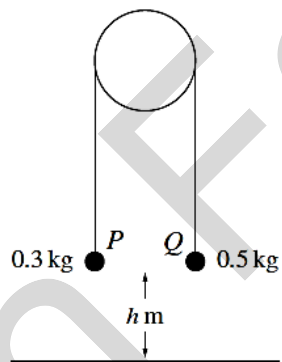
32 Two particles A and B, of masses m kg and 0.3 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley and the particles hang freely below it. The system is released from rest, with both particles 0.8 m above horizontal ground. Particle A reaches the ground with a speed of $0.6\,\text{m}\,\text{s}^{-1}$.

(i) Find the tension in the string during the motion before A reaches the ground. [4]

(ii) Find the value of m. [2]

Answers: (i) 3.07 N (to 3 sf) (ii) $m = 0.314$ (to 3 sf N18/41/Q4

33



Two particles P and Q, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley with the particles hanging freely below it. Q is held at rest with the string taut at a height of h m above a horizontal floor (see diagram). Q is now released and both particles start to move. The pulley is sufficiently high so that P does not reach it at any stage. The time taken for Q to reach the floor is 0.6 s.

(i) Find the acceleration of Q before it reaches the floor and hence find the value of h. [6]

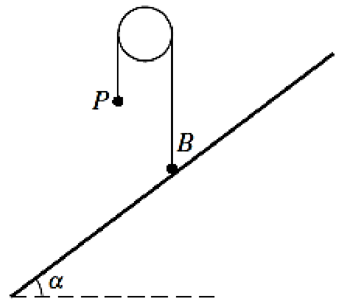
Q remains at rest when it reaches the floor, and P continues to move upwards.

(ii) Find the velocity of P at the instant when Q reaches the floor and the total time taken from the instant at which Q is released until the string becomes taut again. [3]

Answer: $a = 2.5\,\text{ms}^{-2}$ $h = 0.45$ N18/43/Q5

Answer: Velocity of P when Q reaches floor = $1.5\,\text{ms}^{-1}$ Total time = 0.9 s

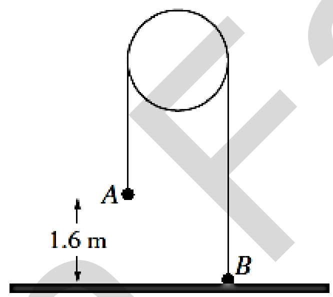
34



A small ball B of mass 4 kg is attached to one end of a light inextensible string. A particle P of mass 3 kg is attached to the other end of the string. The string passes over a fixed smooth pulley. The system is in equilibrium with the string taut and its straight parts vertical. B is at rest on a rough plane inclined to the horizontal at an angle of α, where $\cos \alpha = 0.8$ (see diagram). State the tension in the string and find the normal component of the contact force exerted on B by the plane. [3]

Answer: 30 N 8 N N15/43/Q1

35



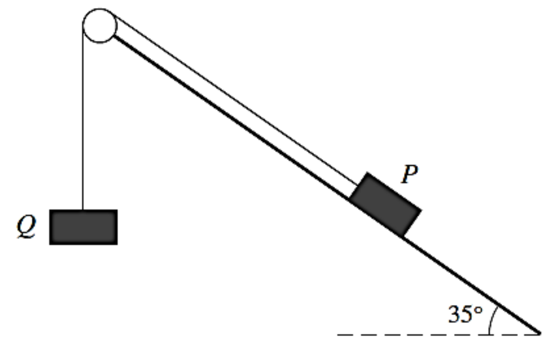
Particles A and B, of masses 0.35 kg and 0.15 kg respectively, are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. The system is at rest with B held on the horizontal floor, the string taut and its straight parts vertical. A is at a height of 1.6 m above the floor (see diagram). B is released and the system begins to move; B does not reach the pulley. Find

(i) the acceleration of the particles and the tension in the string before A reaches the floor, [4]

(ii) the greatest height above the floor reached by B. [3]

Answer: (i) 4 ms^{-2} 2.1 N (ii) 2.24 m N15/43/Q4

36



Blocks P and Q, of mass m kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane inclined at 35° to the horizontal. Block P is at rest on the plane and block Q hangs vertically below the pulley (see diagram). The coefficient of friction between block P and the plane is 0.2. Find the set of values of m for which the two blocks remain at rest. [6]

Answers: The set of values of m for which the two blocks remain at rest is $6.78 \le m \le 12.2$ N15/41/Q4

WORK, ENERGY, POWER

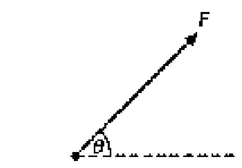
WORK

The product of F and s is referred to as the **work done** by the constant force F as the particle moves through a displacement s.

The S.I. unit of work is the **joule** (J), which is the amount of work done when a force of 1 newton moves an object a distance of 1 metre. The joule is named after the English physicist **James Prescott Joule (1818–89).**

It is important to realise that the displacement must take place in the direction of the force. Hence, any forces that are perpendicular to the displacement do no work.

Frequently, the applied force is directed at an angle to the direction that the displacement occurs. For the object in Example 1 the rope used to pull the object may be inclined to the horizontal.



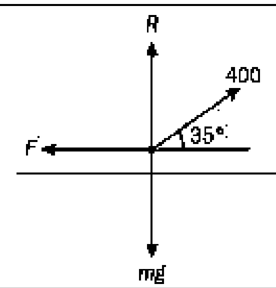
Suppose a force F is applied to an object that is then displaced by a distance s in a direction making an angle θ with the direction of F.

The force F can be resolved into two components: parallel and perpendicular to the direction of the displacement. The perpendicular component, $F\sin\theta$, does no work because there is no displacement in that direction. The parallel component, $F\cos\theta$, is displaced a distance s. Therefore the work done by F is

$$(F\cos\theta) \times s = Fs\cos\theta$$

<u>EXAMPLE 1</u>

A packing case is pulled along horizontal ground a distance of 6 m by means of a rope inclined at 35° to the horizontal. The tension in the rope is 400 N. Find the work done by the tension.



KINETIC ENERGY (KE)

If an object is moving with a speed v the quantity $\frac{1}{2}mv^2$ is called the **kinetic energy** of the body.

So the kinetic energy (KE) of a body is the energy it possesses because of its motion. Equation (1) tells us that when a force does work on a body so as to increase its speed, then the work done is a measure of the increase in the kinetic energy of the body.

EXAMPLE 2

A car of mass 1600 kg is travelling along a straight horizontal road at 15 m s^{-1}. The brakes are applied as the car approaches a junction. The car travels 25 m before coming to rest. Find

a) the initial kinetic energy of the car

b) the work done in stopping the car

c) the force applied in stopping the car.

EXAMPLE 3

A body of mass 6 kg increases its energy by 38 J. If its initial speed was 2 m s^{-1}, find its final speed.

POTENTIAL ENERGY (PE)

If an object increases its height by a distance h the quantity mgh is the increase in **gravitational potential energy** of the object.

Raising an object increases its gravitational potential energy (GPE), while lowering it decreases its GPE. The amount of energy is simply mgh, where h is the distance above some arbitrary reference point. When an object loses height, this potential energy is converted to kinetic energy.

EXAMPLE 4

A carriage on a roller coaster ride has a mass of 130 kg. Find the change in potential energy when the carriage descends 12 m.

EXAMPLE 5

A climber of mass 75 kg scales a mountain 1.8 km high. Find her gain in potential energy.

EXAMPLE 6

A block of mass 2.9 kg slides down the line of greatest slope of a rough inclined plane. The normal reaction between the block and the plane is 2 N.

a) **Show that** if the angle of slope of the plane is θ, then $\cos\theta = \frac{21}{29}$.

b) Calculate the change in gravitational potential energy if the block slides a distance of 2.4 m.

CONSERVATION OF ENERGY

If, as an object moves, all of its potential energy is converted to kinetic energy, then the situation is called a **conservative system.** If some work is done, other than that done by or against gravity, so that not all the potential energy is converted to kinetic energy, this is a **non-conservative system.**

So the total energy of a system remains constant provided no external work is done and there are no sudden changes in the motion of the system.

The total energy of a system remains constant provided no external work is done and there are no sudden changes in the motion of the system. This is known as the **principle of conservation of energy**.

EXAMPLE 7

A ball of mass 0.6 kg is dropped from rest at a height of 10 m above the ground. Neglecting air resistance, find

a) the loss in gravitational potential energy in falling to the ground

b) the gain in kinetic energy at the instant the ball reaches the ground

c) the speed with which the ball hits the ground.

EXAMPLE 8

A smooth slope is inclined at $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. A particle of mass 0.4 kg is released from rest at the top of the slope. The particle reaches the bottom of the slope at a speed of $8\ \mathrm{m\,s^{-1}}$. Find the length of the slope.

THE WORK-ENERGY PRINCIPLE

The total work done on any system is equal to the total change in energy. This is known as the **work–energy principle.**

As an object moves, if work is done, and so not all of the potential energy is converted to kinetic energy, the system is non-conservative (see Section 7.4). In these situations we can still make use of energy to solve a problem because of the work–energy principle.

EXAMPLE 9

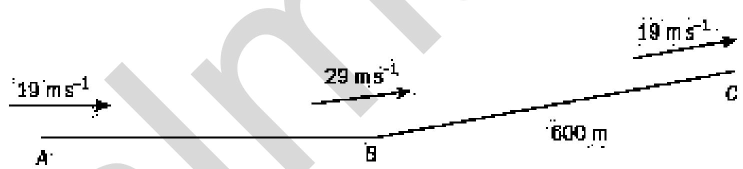
A particle of mass 4 kg is projected down a plane inclined at 30° to the horizontal at a speed of $1\ m\ s^{-1}$. There is a constant resistance of 5 N. Find the speed of the particle after it has travelled 6 m down the plane.

EXAMPLE 10

A car of mass 1200 kg has a speed of $26\ m\ s^{-1}$ at the bottom of a hill inclined at 5° to the horizontal. The car travels up the line of greatest slope of the hill. After a distance of 500 m the car's speed has decreased to $12\ m\ s^{-1}$. The resistance to motion is constant and has magnitude 400 N. Find the constant driving force produced by the car's engine.

There are cases where the energy done by a specific force is given. The following example shows how to deal with this type of problem. It cannot be assumed that the force is constant and hence the use of constant acceleration formulae is prohibited.

Example 11



A car of mass 1250 kg travels along a road that has a straight horizontal section *AB* and a straight inclined section *BC*. The length of *BC* is 600 m. The speed of the car at *A*, *B* and *C* is $19\ m\ s^{-1}$, $29\ m\ s^{-1}$ and $19\ m\ s^{-1}$, respectively (see diagram).

a) The work done against the resistance to motion of the car, as it travels from *A* to *B*, is 525 kJ. Find the work done by the driving force as the car travels from *A* to *B*.

b) As the car travels from B to C, the resistance to motion is 450 N and the work done by the driving force is 370 kJ. Find the height of C above the level of AB.

Examination advice

It is tempting to use the constant acceleration equations in this question. However, this would require the assumption that the driving force and the resistance are constant, which is not stated in the question. Only the work done by the resistance and by the driving force are given. This is something to be aware of in questions of this type.

EXAMPLE 11

In a playground a small child of mass 20 kg goes down a slide, as shown in the diagram. The slide is 1.5 m high and there is a constant resistance of 10 N between the child and the slide. The total length of the slide is 3.5 m. The child starts from rest at the top of the slide. Find

a) the loss in gravitational energy of the child in sliding to the bottom of the slide

b) the gain in kinetic energy at the instant she reaches the end of the slide

c) her speed at the end of the slide.

POWER

The rate at which work is done is called power, which can be expressed in a formula.

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

One unit of power is produced when work is done at the rate of one joule per second. This unit is called the watt, W, after **James Watt (1736–1819)**, a Scottish inventor and engineer best known for his work on steam engine development. The watt is a relatively small unit, so it is often useful to use the kilowatt (kW), where 1 kW = 1000 W.

For example, if it takes a car 50 seconds to move 100 m when the average force produced by the engine is 300 N, then the average rate of working or average power is

$$\frac{\text{work done}}{\text{time taken}} = \frac{300 \times 100}{50} = 600\,\text{W}$$

In some cases, when the power is specified, the speed is required at a given time. A car that is moved a distance of s metres in t seconds by a driving force F newtons has

$$\text{power} = \frac{F \times s}{t} = F \times \frac{s}{t}$$

But the quantity $\frac{s}{t}$ is the speed, $v\,\text{m}\,\text{s}^{-1}$, of the car.

Hence

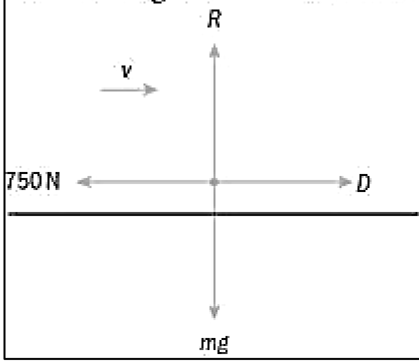
$$\text{power} = Fv$$

If the speed is not constant the value of Fv gives the power at the instant when the speed is $v\,\text{m}\,\text{s}^{-1}$.

There is obviously a limit to the power that a car or any other vehicle can produce. When the maximum power is attained the speed produced is also a maximum. In this case there is no acceleration possible, and the resultant force is zero.

__EXAMPLE 12__

A car moves along a horizontal road against a resistance of 750 N. The maximum power of the car's engine is 12 kW. Determine the maximum speed of the car.



__EXAMPLE 13__

A mass of 164 kg is raised, by a crane, vertically upwards through a distance of 35 m in 106 seconds. Calculate the average power of the crane.

ACCELERATION AND VARIABLE RESISTANCE

If, at a particular instant, a vehicle exerts more driving force than the total resistive forces there will be a resultant force in the direction of motion. In this case the vehicle will accelerate. The acceleration can be found by applying Newton's second law.

Be aware that the acceleration will be different at different instants. This is because, if the power remains constant, the vehicle accelerates, changing its speed, and hence the driving force will change. So we can only calculate acceleration at a particular instant in time.

__EXAMPLE 14__

A car travels along a horizontal straight road against a constant resistive force of magnitude 275 N. The mass of the car is 1400 kg and its engine is working at a rate of 7.5 kW. Calculate

a) the acceleration at the instant when the car has a speed of 12 m s^{-1}

b) the speed of the car at the instant when it is accelerating at a rate of 0.1 m s^{-2} up the line of greatest slope of a hill inclined at an angle of 5° to the horizontal.

EXAMPLE 15

A car of mass 1250 kg travels on a straight horizontal road. It experiences a resistive force of magnitude $25v$ N, where v m s^{-1} is the car's speed. The maximum speed of the car on this road is 60 m s^{-1}. Calculate

a) the car's maximum power

b) the car's maximum possible acceleration when its speed is 30 m s^{-1}.

EXAMPLE 16

A car of mass 1000 kg moves along a horizontal straight road, passing through points A and B. The power of its engine is constant and equal to 15 000 W. The driving force exerted by the engine is 750 N at A and 500 N at B. Find the speed of the car at A and at B, and hence find the increase in the car's kinetic energy as it moves from A to B.

[Cambridge AS and A Level Mathematics 9709, Paper 41 Q1 November 2009]

EXAMPLE 17

A car of mass 1150 kg travels up a straight hill inclined at 1.2° to the horizontal. The resistance to motion of the car is 975 N. Find the acceleration of the car at an instant when it is moving with speed 16 m s^{-1} and the engine is working at a power of 35 kW.

[Cambridge AS and A Level Mathematics 9709, Paper 41 Q1 June 2010]

EXAMPLE 18

A car of mass 1200 kg travels along a horizontal straight road. The power provided by the car's engine is constant and equal to 20 kW. The resistance to the car's motion is constant and equal to 500 N. The car passes through the points A and B with speeds 10 m s^{-1} and 25 m s^{-1} respectively. The car takes 30.5 s to travel from A to B.

(i) Find the acceleration of the car at A.

(ii) By considering work and energy, find the distance AB.

[Cambridge AS and A Level Mathematics 9709, Paper 4 Q7 June 2005]

SELECTED PAST PAPER QUESTIONS

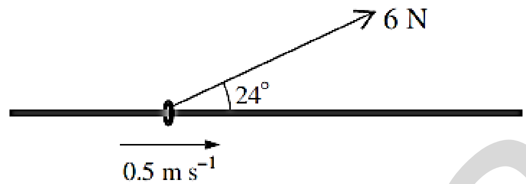
QUESTION 1

J12/41/Q1

A car of mass 880 kg travels along a straight horizontal road with its engine working at a constant rate of P W. The resistance to motion is 700 N. At an instant when the car's speed is $16\,\text{m s}^{-1}$ its acceleration is $0.625\,\text{m s}^{-2}$. Find the value of P. [4]

QUESTION 2

J12/43/Q1



A ring is threaded on a fixed horizontal bar. The ring is attached to one end of a light inextensible string which is used to pull the ring along the bar at a constant speed of $0.5\,\text{m s}^{-1}$. The string makes a constant angle of 24° with the bar and the tension in the string is 6 N (see diagram). Find the work done by the tension in a period of 8 s. [3]

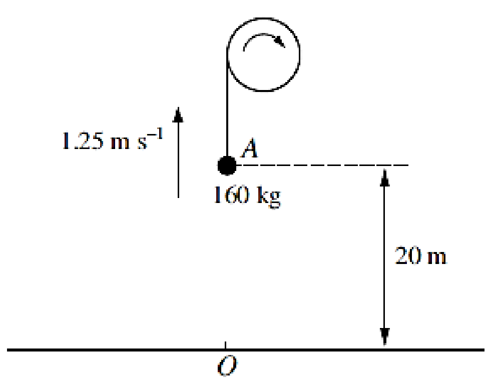
QUESTION 3

N12/42/Q1

A block is pushed along a horizontal floor by a force of magnitude 45 N acting at an angle of 14° to the horizontal (see diagram). Find the work done by the force in moving the block a distance of 25 m. [3]

QUESTION 4

J12/41/Q3



A load of mass 160 kg is pulled vertically upwards, from rest at a fixed point O on the ground, using a winding drum. The load passes through a point A, 20 m above O, with a speed of 1.25 m s^{-1} (see diagram). Find, for the motion from O to A,

(i) the gain in the potential energy of the load, [1]

(ii) the gain in the kinetic energy of the load. [2]

The power output of the winding drum is constant while the load is in motion.

(iii) Given that the work done against the resistance to motion from O to A is 20 kJ and that the time taken for the load to travel from O to A is 41.7 s, find the power output of the winding drum. [3]

QUESTION 5

J12/43/Q5

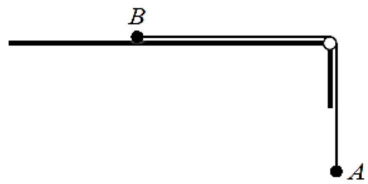
A lorry of mass 16 000 kg moves on a straight hill inclined at angle $\alpha°$ to the horizontal. The length of the hill is 500 m.

(i) While the lorry moves from the bottom to the top of the hill at constant speed, the resisting force acting on the lorry is 800 N and the work done by the driving force is 2800 kJ. Find the value of α. [4]

(ii) On the return journey the speed of the lorry is 20 m s^{-1} at the top of the hill. While the lorry travels down the hill, the work done by the driving force is 2400 kJ and the work done against the resistance to motion is 800 kJ. Find the speed of the lorry at the bottom of the hill. [4]

QUESTION 6

N14/42/Q5



Particles A and B, each of mass 0.3 kg, are connected by a light inextensible string. The string passes over a small smooth pulley fixed at the edge of a rough horizontal surface. Particle A hangs freely and particle B is held at rest in contact with the surface (see diagram). The coefficient of friction between B and the surface is 0.7. Particle B is released and moves on the surface without reaching the pulley.

(i) Find, for the first 0.9 m of B's motion,

(a) the work done against the frictional force acting on B, [2]

(b) the loss of potential energy of the system, [1]

(c) the gain in kinetic energy of the system. [2]

At the instant when B has moved 0.9 m the string breaks. A is at a height of 0.54 m above a horizontal floor at this instant.

(ii) Find the speed with which A reaches the floor. [3]

HOMEWORK: WORK, POWER, ENERGY VARIANT 42

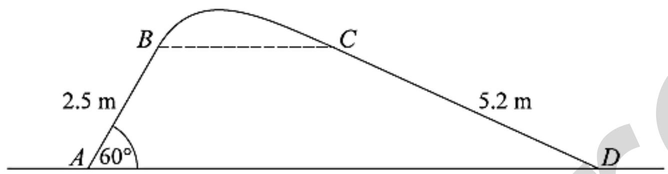
1 A crate of mass 800 kg is lifted vertically, at constant speed, by the cable of a crane. Find

(i) the tension in the cable, [1]

(ii) the power applied to the crate in increasing the height by 20 m in 50 s. [3]

Answers: **(i)** 8000 N; **(ii)** 3200 W. **J03/Q1**

2



The diagram shows a vertical cross-section *ABCD* of a surface. The parts *AB* and *CD* are straight and have lengths 2.5 m and 5.2 m respectively. *AD* is horizontal, and *AB* is inclined at 60° to the horizontal. The points *B* and *C* are at the same height above *AD*. The parts of the surface containing *AB* and *BC* are smooth. A particle *P* is given a velocity of $8\ \text{m s}^{-1}$ at *A*, in the direction *AB*, and it subsequently reaches *D*. The particle does not lose contact with the surface during this motion.

(i) Find the speed of *P* at *B*. [4]

(ii) Show that the maximum height of the cross-section, above *AD*, is less than 3.2 m. [2]

(iii) State briefly why *P*'s speed at *C* is the same as its speed at *B*. [1]

(iv) The frictional force acting on the particle as it travels from *C* to *D* is 1.4 N. Given that the mass of *P* is 0.4 kg, find the speed with which *P* reaches *D*. [4]

Answers: (i) $4.55\ \text{ms}^{-1}$; (iii) Path *BC* is smooth and *B* and *C* are at the same height (⇒ KE the same at *B* and *C*); (iv) $5.25\ \text{ms}^{-1}$. **J03/Q7**

3 The top of an inclined plane is at a height of 0.7 m above the bottom. A block of mass 0.2 kg is released from rest at the top of the plane and slides a distance of 2.5 m to the bottom. Find the kinetic energy of the block when it reaches the bottom of the plane in each of the following cases:

(i) the plane is smooth, [2]

(ii) the coefficient of friction between the plane and the block is 0.15. [5]

Answers: (i) 1.4 J; (ii) 0.68 J. **J04/Q4**

4 A car of mass 1200 kg travels along a horizontal straight road. The power of the car's engine is 20 kW. The resistance to the car's motion is 400 N.

(i) Find the speed of the car at an instant when its acceleration is $0.5\ \text{m s}^{-2}$. [4]

(ii) Show that the maximum possible speed of the car is $50\ \text{m s}^{-1}$. [2]

The work done by the car's engine as the car travels from a point A to a point B is 1500 kJ.

(iii) Given that the car is travelling at its maximum possible speed between A and B, find the time taken to travel from A to B. [2]

Answers: (i) $20\ \text{ms}^{-1}$; (iii) 75 s. **J04/Q6**

5 A small block is pulled along a rough horizontal floor at a constant speed of $1.5\ \text{m s}^{-1}$ by a constant force of magnitude 30 N acting at an angle of $\theta°$ upwards from the horizontal. Given that the work done by the force in 20 s is 720 J, calculate the value of θ. [3]

Answer: 36.9. **J05/Q1**

6 A car of mass 1200 kg travels along a horizontal straight road. The power provided by the car's engine is constant and equal to 20 kW. The resistance to the car's motion is constant and equal to 500 N. The car passes through the points A and B with speeds $10\ \text{m s}^{-1}$ and $25\ \text{m s}^{-1}$ respectively. The car takes 30.5 s to travel from A to B.

(i) Find the acceleration of the car at A. [4]

(ii) By considering work and energy, find the distance AB. [8]

Answers: (i) $1.25\ \text{ms}^{-2}$; (ii) 590 m. **J05/Q7**

7 A block of mass 50 kg is pulled up a straight hill and passes through points A and B with speeds $7\ \text{m s}^{-1}$ and $3\ \text{m s}^{-1}$ respectively. The distance AB is 200 m and B is 15 m higher than A. For the motion of the block from A to B, find

(i) the loss in kinetic energy of the block, [2]

(ii) the gain in potential energy of the block. [2]

The resistance to motion of the block has magnitude 7.5 N.

(iii) Find the work done by the pulling force acting on the block. [2]

The pulling force acting on the block has constant magnitude 45 N and acts at an angle $\alpha°$ upwards from the hill.

(iv) Find the value of α. [3]

Answers: (i) 1000 J; (ii) 7500 J; (iii) 8000 J; (iv) 27.3. **J06/Q6**

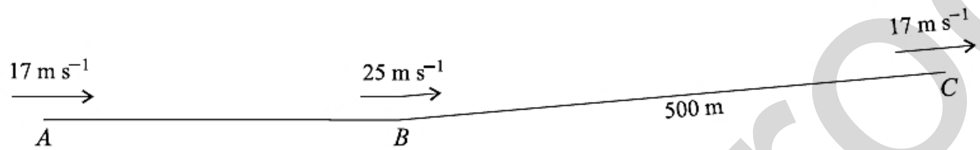
8 A car travels along a horizontal straight road with increasing speed until it reaches its maximum speed of $30\,\mathrm{m\,s^{-1}}$. The resistance to motion is constant and equal to R N, and the power provided by the car's engine is 18 kW.

(i) Find the value of R. [3]

(ii) Given that the car has mass 1200 kg, find its acceleration at the instant when its speed is $20\,\mathrm{m\,s^{-1}}$. [3]

Answers: (i) 600; (ii) $0.25\ \mathrm{ms^{-2}}$. **J07/Q3**

9



A lorry of mass 12 500 kg travels along a road that has a straight horizontal section AB and a straight inclined section BC. The length of BC is 500 m. The speeds of the lorry at A, B and C are $17\,\mathrm{m\,s^{-1}}$, $25\,\mathrm{m\,s^{-1}}$ and $17\,\mathrm{m\,s^{-1}}$ respectively (see diagram).

(i) The work done against the resistance to motion of the lorry, as it travels from A to B, is 5000 kJ. Find the work done by the driving force as the lorry travels from A to B. [4]

(ii) As the lorry travels from B to C, the resistance to motion is 4800 N and the work done by the driving force is 3300 kJ. Find the height of C above the level of AB. [4]

Answers: (i) 7100 kJ; (ii) 24 m. **J07/Q5**

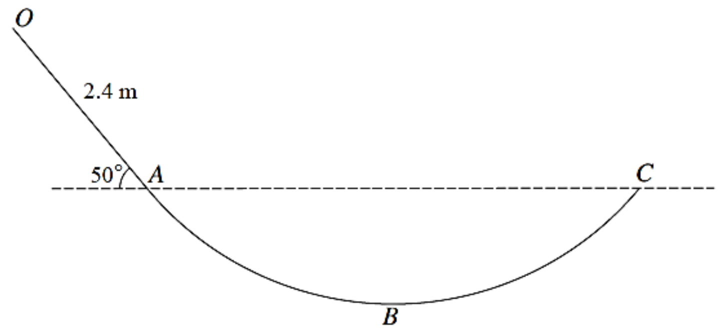
10 A block is being pulled along a horizontal floor by a rope inclined at 20° to the horizontal. The tension in the rope is 851 N and the block moves at a constant speed of $2.5\,\mathrm{m\,s^{-1}}$.

(i) Show that the work done on the block in 12 s is approximately 24 kJ. [3]

(ii) Hence find the power being applied to the block, giving your answer to the nearest kW. [1]

Answer: (ii) 2 kW. **J08/Q2**

11



$OABC$ is a vertical cross-section of a smooth surface. The straight part OA has length 2.4 m and makes an angle of 50° with the horizontal. A and C are at the same horizontal level and B is the lowest point of the cross-section (see diagram). A particle P of mass 0.8 kg is released from rest at O and moves on the surface. P remains in contact with the surface until it leaves the surface at C. Find

(i) the kinetic energy of P at A, [2]

(ii) the speed of P at C. [2]

The greatest speed of P is $8\,\text{m s}^{-1}$.

(iii) Find the depth of B below the horizontal through A and C. [3]

Answers: (i) 14.7 J; (ii) $6.06\,\text{ms}^{-1}$; (iii) 1.36 m. **J08/Q4**

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12 A particle P of mass 0.6 kg is projected vertically upwards with speed $5.2\,\text{m s}^{-1}$ from a point O which is 6.2 m above the ground. Air resistance acts on P so that its deceleration is $10.4\,\text{m s}^{-2}$ when P is moving upwards, and its acceleration is $9.6\,\text{m s}^{-2}$ when P is moving downwards. Find

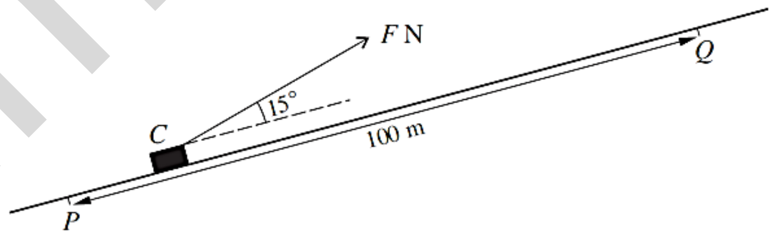
(i) the greatest height above the ground reached by P, [3]

(ii) the speed with which P reaches the ground, [2]

(iii) the total work done on P by the air resistance. [4]

Answers: (i) 7.5 m; (ii) $12\,\text{ms}^{-1}$; (iii) 2.11 J. **J08/Q6**

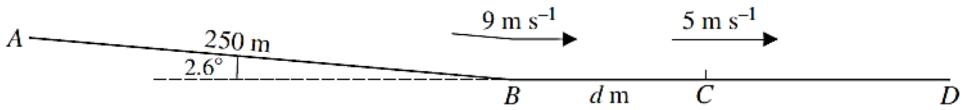
13



A crate C is pulled at constant speed up a straight inclined path by a constant force of magnitude F N, acting upwards at an angle of 15° to the path. C passes through points P and Q which are 100 m apart (see diagram). As C travels from P to Q the work done against the resistance to C's motion is 900 J, and the gain in C's potential energy is 2100 J. Write down the work done by the pulling force as C travels from P to Q, and hence find the value of F. [3]

Answers: 3000 J, 31.1. **J09/Q2**

16



A cyclist and his machine have a total mass of 80 kg. The cyclist starts from rest at the top A of a straight path AB, and freewheels (moves without pedalling or braking) down the path to B. The path AB is inclined at 2.6° to the horizontal and is of length 250 m (see diagram).

(i) Given that the cyclist passes through B with speed 9 m s^{-1}, find the gain in kinetic energy and the loss in potential energy of the cyclist and his machine. Hence find the work done against the resistance to motion of the cyclist and his machine. [3]

The cyclist continues to freewheel along a horizontal straight path BD until he reaches the point C, where the distance BC is d m. His speed at C is 5 m s^{-1}. The resistance to motion is constant, and is the same on BD as on AB.

(ii) Find the value of d. [3]

The cyclist starts to pedal at C, generating 425 W of power.

(iii) Find the acceleration of the cyclist immediately after passing through C. [3]

Answers: (i) 3240 J, 9070 J, 5830 J; (ii) 96.0; (iii) 0.771 ms^{-2}. **J09/Q5**

17 A car of mass 1150 kg travels up a straight hill inclined at 1.2° to the horizontal. The resistance to motion of the car is 975 N. Find the acceleration of the car at an instant when it is moving with speed 16 m s^{-1} and the engine is working at a power of 35 kW. [4]

Answers: (i) 20 s; (ii) 80 s; (iii) 4 ms^{-1}; (iv) 1170 m. **J10/Q1**

18 P and Q are fixed points on a line of greatest slope of an inclined plane. The point Q is at a height of 0.45 m above the level of P. A particle of mass 0.3 kg moves upwards along the line PQ.

(i) Given that the plane is smooth and that the particle just reaches Q, find the speed with which it passes through P. [3]

(ii) It is given instead that the plane is rough. The particle passes through P with the same speed as that found in part **(i)**, and just reaches a point R which is between P and Q. The work done against the frictional force in moving from P to R is 0.39 J. Find the potential energy gained by the particle in moving from P to R and hence find the height of R above the level of P. [4]

Answer: (i) 3 m^{-1}; (ii) 0.96 J, 0.32 m. **J10/Q5**

19 A car of mass 1000 kg travels along a horizontal straight road with its engine working at a constant rate of 20 kW. The resistance to motion of the car is 600 N. Find the acceleration of the car at an instant when its speed is 25 m s^{-1}. [3]

Answer: 0.2 ms^{-2}. **N02/Q1**

20 **(i)** A particle P of mass 1.2 kg is released from rest at the top of a slope and starts to move. The slope has length 4 m and is inclined at 25° to the horizontal. The coefficient of friction between P and the slope is $\frac{1}{4}$. Find

(a) the frictional component of the contact force on P, [2]

(b) the acceleration of P, [2]

(c) the speed with which P reaches the bottom of the slope. [2]

(ii) After reaching the bottom of the slope, P moves freely under gravity and subsequently hits a horizontal floor which is 3 m below the bottom of the slope.

(a) Find the loss in gravitational potential energy of P during its motion from the bottom of the slope until it hits the floor. [1]

(b) Find the speed with which P hits the floor. [3]

Answers: **(i)(a)** 2.72 N, **(b)** 1.96ms^{-2}, **(c)** 3.96 ms^{-1}; **(ii)(a)** 36 J, **(b)** 8.70 ms^{-1}. **N02/Q6**

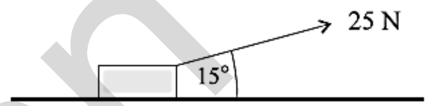
21 A motorcycle of mass 100 kg is travelling on a horizontal straight road. Its engine is working at a rate of 8 kW. At an instant when the speed of the motorcycle is 25 m s^{-1} its acceleration is 0.5 m s^{-2}. Find, at this instant,

(i) the force produced by the engine, [1]

(ii) the resistance to motion of the motorcycle. [3]

Answers: **(i)** 320 N; **(ii)** 270 N. **N03/Q1**

22



A crate of mass 3 kg is pulled at constant speed along a horizontal floor. The pulling force has magnitude 25 N and acts at an angle of 15° to the horizontal, as shown in the diagram. Find

(i) the work done by the pulling force in moving the crate a distance of 2 m, [2]

(ii) the normal component of the contact force on the crate. [3]

Answers: **(i)** 48.3 J; **(ii)** 23.5 N. **N03/Q3**

23 A car of mass 1250 kg travels down a straight hill with the engine working at a power of 22 kW. The hill is inclined at 3° to the horizontal and the resistance to motion of the car is 1130 N. Find the speed of the car at an instant when its acceleration is 0.2 m s^{-2}. [5]

Answer: 30.3 ms^{-1}. **N04/Q3**

24 A lorry of mass 16 000 kg climbs from the bottom to the top of a straight hill of length 1000 m at a constant speed of $10\,\mathrm{m\,s^{-1}}$. The top of the hill is 20 m above the level of the bottom of the hill. The driving force of the lorry is constant and equal to 5000 N. Find

(i) the gain in gravitational potential energy of the lorry, [1]

(ii) the work done by the driving force, [1]

(iii) the work done against the force resisting the motion of the lorry. [1]

On reaching the top of the hill the lorry continues along a straight horizontal road against a constant resistance of 1500 N. The driving force of the lorry is not now constant, and the speed of the lorry increases from $10\,\mathrm{m\,s^{-1}}$ at the top of the hill to $25\,\mathrm{m\,s^{-1}}$ at the point P. The distance of P from the top of the hill is 2000 m.

(iv) Find the work done by the driving force of the lorry while the lorry travels from the top of the hill to P. [5]

Answers: (i) 3200 kJ; (ii) 5000 kJ; (iii) 1800 kJ; (iv) 7200 kJ. **N04/Q4**

25 A crate of mass 50 kg is dragged along a horizontal floor by a constant force of magnitude 400 N acting at an angle $\alpha°$ upwards from the horizontal. The total resistance to motion of the crate has constant magnitude 250 N. The crate starts from rest at the point O and passes the point P with a speed of $2\,\mathrm{m\,s^{-1}}$. The distance OP is 20 m. For the crate's motion from O to P, find

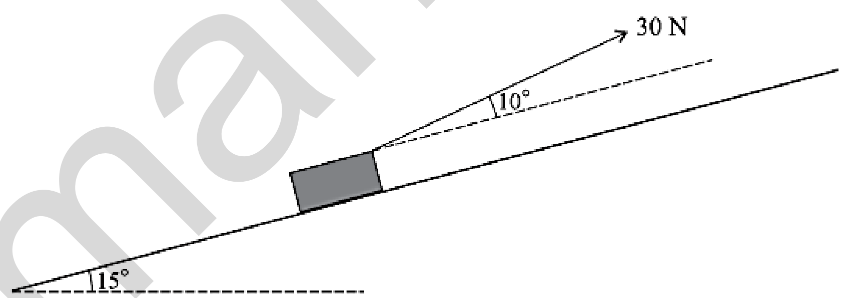
(i) the increase in kinetic energy of the crate, [1]

(ii) the work done against the resistance to the motion of the crate, [1]

(iii) the value of α. [3]

N05/Q2

26



A box of mass 8 kg is pulled, at constant speed, up a straight path which is inclined at an angle of 15° to the horizontal. The pulling force is constant, of magnitude 30 N, and acts upwards at an angle of 10° from the path (see diagram). The box passes through the points A and B, where $AB = 20$ m and B is above the level of A. For the motion from A to B, find

(i) the work done by the pulling force, [2]

(ii) the gain in potential energy of the box, [2]

(iii) the work done against the resistance to motion of the box. [1]

Answers: (i) 591 J; (ii) 414 J; (iii) 177 J. **N06/Q1**

27 A cyclist travels along a straight road working at a constant rate of 420 W. The total mass of the cyclist and her cycle is 75 kg. Ignoring any resistance to motion, find the acceleration of the cyclist at an instant when she is travelling at $5\,\mathrm{m\,s^{-1}}$,

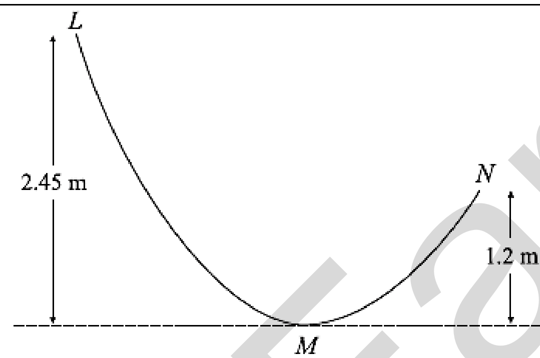
(i) given that the road is horizontal,

(ii) given instead that the road is inclined at $1.5°$ to the horizontal and the cyclist is travelling up the slope.

[5]

Answers: **(i)** $1.12\ \mathrm{ms^{-2}}$; **(ii)** $0.858\ \mathrm{ms^{-2}}$. **N06/Q3**

28



The diagram shows the vertical cross-section LMN of a fixed smooth surface. M is the lowest point of the cross-section. L is 2.45 m above the level of M, and N is 1.2 m above the level of M. A particle of mass 0.5 kg is released from rest at L and moves on the surface until it leaves it at N. Find

(i) the greatest speed of the particle, [3]

(ii) the kinetic energy of the particle at N. [2]

The particle is now projected from N, with speed $v\,\mathrm{m\,s^{-1}}$, along the surface towards M.

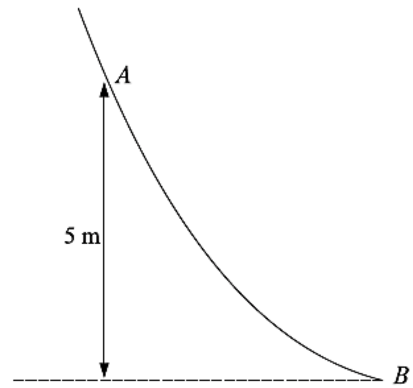
(iii) Find the least value of v for which the particle will reach L. [2]

Answers: **(i)** $7\ \mathrm{ms^{-1}}$; **(ii)** 6.25 J; **(iii)** 5. **N06/Q5**

29 A car of mass 900 kg travels along a horizontal straight road with its engine working at a constant rate of P kW. The resistance to motion of the car is 550 N. Given that the acceleration of the car is $0.2\,\mathrm{m\,s^{-2}}$ at an instant when its speed is $30\,\mathrm{m\,s^{-1}}$, find the value of P. [4]

Answer: 21.9. **N07/Q1**

30



The diagram shows the vertical cross-section of a surface. A and B are two points on the cross-section, and A is 5 m higher than B. A particle of mass 0.35 kg passes through A with speed 7 m s^{-1}, moving on the surface towards B.

(i) Assuming that there is no resistance to motion, find the speed with which the particle reaches B. [3]

(ii) Assuming instead that there is a resistance to motion, and that the particle reaches B with speed 11 m s^{-1}, find the work done against this resistance as the particle moves from A to B. [3]

Answers: (i) 12.2 ms^{-1}; (ii) 4.9 J. **N07/Q4**

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31 A car of mass 1200 kg is travelling on a horizontal straight road and passes through a point A with speed 25 m s^{-1}. The power of the car's engine is 18 kW and the resistance to the car's motion is 900 N.

(i) Find the deceleration of the car at A. [4]

(ii) Show that the speed of the car does not fall below 20 m s^{-1} while the car continues to move with the engine exerting a constant power of 18 kW. [2]

Answer: (i) 0.15 ms^{-2}. **N08/Q3**

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32 A load of mass 160 kg is lifted vertically by a crane, with constant acceleration. The load starts from rest at the point O. After 7 s, it passes through the point A with speed 0.5 m s^{-1}. By considering energy, find the work done by the crane in moving the load from O to A. [6]

Answer: 2820 J. **N08/Q4**

33 A lorry of mass 15 000 kg moves with constant speed 14 m s^{-1} from the top to the bottom of a straight hill of length 900 m. The top of the hill is 18 m above the level of the bottom of the hill. The total work done by the resistive forces acting on the lorry, including the braking force, is 4.8×10^6 J. Find

(i) the loss in gravitational potential energy of the lorry, [1]

(ii) the work done by the driving force. [1]

On reaching the bottom of the hill the lorry continues along a straight horizontal road against a constant resistance of 1600 N. There is no braking force acting. The speed of the lorry increases from 14 m s^{-1} at the bottom of the hill to 16 m s^{-1} at the point X, where X is 2500 m from the bottom of the hill.

(iii) By considering energy, find the work done by the driving force of the lorry while it travels from the bottom of the hill to X. [3]

Answers: (i) 2.7×10^6 J; (ii) 2.1×10^6 J; (iii) 4.45×10^6 J. **N09/42/Q2**

33 A car of mass 1250 kg travels along a horizontal straight road with increasing speed. The power provided by the car's engine is constant and equal to 24 kW. The resistance to the car's motion is constant and equal to 600 N.

(i) Show that the speed of the car cannot exceed 40 m s^{-1}. [3]

(ii) Find the acceleration of the car at an instant when its speed is 15 m s^{-1}. [3]

N09/42/Q3

Answer: (ii) 0.8 ms^{-2}.

34 A cyclist, working at a constant rate of 400 W, travels along a straight road which is inclined at 2° to the horizontal. The total mass of the cyclist and his cycle is 80 kg. Ignoring any resistance to motion, find, correct to 1 decimal place, the acceleration of the cyclist when he is travelling

(i) uphill at 4 m s^{-1},

(ii) downhill at 4 m s^{-1}.

[5]

N10/42/Q2

35 A block of mass 20 kg is pulled from the bottom to the top of a slope. The slope has length 10 m and is inclined at 4.5° to the horizontal. The speed of the block is 2.5 m s^{-1} at the bottom of the slope and 1.5 m s^{-1} at the top of the slope.

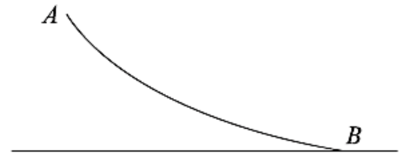
(i) Find the loss of kinetic energy and the gain in potential energy of the block. [3]

(ii) Given that the work done against the resistance to motion is 50 J, find the work done by the pulling force acting on the block. [2]

(iii) Given also that the pulling force is constant and acts at an angle of 15° upwards from the slope, find its magnitude. [2]

N10/42/Q4

36



The diagram shows a vertical cross-section of a surface. A and B are two points on the cross-section. A particle of mass 0.15 kg is released from rest at A.

(i) Assuming that the particle reaches B with a speed of $8\,\text{m s}^{-1}$ and that there are no resistances to motion, find the height of A above B. [3]

(ii) Assuming instead that the particle reaches B with a speed of $6\,\text{m s}^{-1}$ and that the height of A above B is 4 m, find the work done against the resistances to motion. [3]

Answers: **(i)** 3.2 m; **(ii)** 3.3 J. **N03/Q4**

37 A load is pulled along horizontal ground for a distance of 76 m, using a rope. The rope is inclined at 5° above the horizontal and the tension in the rope is 65 N.

(i) Find the work done by the tension. [2]

At an instant during the motion the velocity of the load is $1.5\,\text{m s}^{-1}$.

(ii) Find the rate of working of the tension at this instant. [2]

Answers: (i) 4920 J; (ii) 97.1 W. **J11/42/Q1**

38 An object of mass 8 kg slides down a line of greatest slope of an inclined plane. Its initial speed at the top of the plane is $3\,\text{m s}^{-1}$ and its speed at the bottom of the plane is $8\,\text{m s}^{-1}$. The work done against the resistance to motion of the object is 120 J. Find the height of the top of the plane above the level of the bottom. [4]

Answer: 4.25 m. **J11/42/Q2**

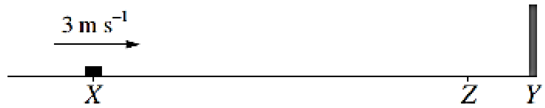
39 A car of mass 1250 kg travels from the bottom to the top of a straight hill which has length 400 m and is inclined to the horizontal at an angle of α, where $\sin\alpha = 0.125$. The resistance to the car's motion is 800 N. Find the work done by the car's engine in each of the following cases.

(i) The car's speed is constant. [4]

(ii) The car's initial speed is $6\,\text{m s}^{-1}$, the car's driving force is 3 times greater at the top of the hill than it is at the bottom, and the car's power output is 5 times greater at the top of the hill than it is at the bottom. [5]

Answers: **(i)** Work done = 945 000 J **(ii)** Work done = 985 000 J **J12/42/Q6**

40



The frictional force acting on a small block of mass 0.15 kg, while it is moving on a horizontal surface, has magnitude 0.12 N. The block is set in motion from a point X on the surface, with speed $3\ \mathrm{m\,s^{-1}}$. It hits a vertical wall at a point Y on the surface 2 s later. The block rebounds from the wall and moves directly towards X before coming to rest at the point Z (see diagram). At the instant that the block hits the wall it loses 0.072 J of its kinetic energy. The velocity of the block, in the direction from X to Y, is $v\ \mathrm{m\,s^{-1}}$ at time t s after it leaves X.

(i) Find the values of v when the block arrives at Y and when it leaves Y, and find also the value of t when the block comes to rest at Z. Sketch the velocity-time graph. [9]

(ii) The displacement of the block from X, in the direction from X to Y, is s m at time t s. Sketch the displacement-time graph. Show on your graph the values of s and t when the block is at Y and when it comes to rest at Z. [4]

Answers: (i) $v_{Approach} = 1.4\ \mathrm{ms^{-1}}$, $v_{Return} = -1.0\ \mathrm{ms^{-1}}$, $t = 3.25$ s **J12/42/Q7**

41 A and B are two points 50 metres apart on a straight path inclined at an angle θ to the horizontal, where $\sin\theta = 0.05$, with A above the level of B. A block of mass 16 kg is pulled down the path from A to B. The block starts from rest at A and reaches B with a speed of $10\ \mathrm{m\,s^{-1}}$. The work done by the pulling force acting on the block is 1150 J.

(i) Find the work done against the resistance to motion. [3]

The block is now pulled up the path from B to A. The work done by the pulling force and the work done against the resistance to motion are the same as in the case of the downward motion.

(ii) Show that the speed of the block when it reaches A is the same as its speed when it started at B. [2]

Answer: (i) 750J **J13/42/Q2**

42 A car of mass 1000 kg is travelling on a straight horizontal road. The power of its engine is constant and equal to P kW. The resistance to motion of the car is 600 N. At an instant when the car's speed is $25\ \mathrm{m\,s^{-1}}$, its acceleration is $0.2\ \mathrm{m\,s^{-2}}$. Find

(i) the value of P, [4]

(ii) the steady speed at which the car can travel. [3]

Answers: (i) $P = 20$ (ii) Steady speed = $33.3\ \mathrm{ms^{-1}}$ **J13/42/Q5**

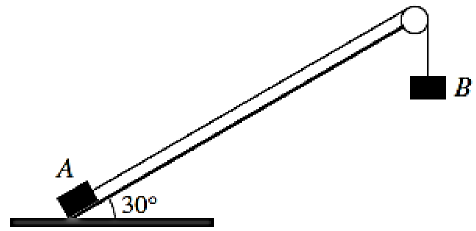
43 A car of mass 600 kg travels along a straight horizontal road. The resistance to the car's motion is constant and equal to R N.

(i) Find the value of R, given that the car's acceleration is $1.4\ \mathrm{m\,s^{-2}}$ at an instant when the car's speed is $18\ \mathrm{m\,s^{-1}}$ and its engine is working at a rate of 22.5 kW. [4]

(ii) Find the rate of working of the car's engine when the car is moving with a constant speed of $15\ \mathrm{m\,s^{-1}}$. [1]

Answer: $R = 410$ *Answer*: Rate of working of car's engine = 6150 W **J14/42/Q1**

44



A light inextensible rope has a block A of mass 5 kg attached at one end, and a block B of mass 16 kg attached at the other end. The rope passes over a smooth pulley which is fixed at the top of a rough plane inclined at an angle of 30° to the horizontal. Block A is held at rest at the bottom of the plane and block B hangs below the pulley (see diagram). The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Block A is released from rest and the system starts to move. When each of the blocks has moved a distance of x m each has speed $v\ \mathrm{m\,s^{-1}}$.

(i) Write down the gain in kinetic energy of the system in terms of v. [1]

(ii) Find, in terms of x,

(a) the loss of gravitational potential energy of the system, [2]

(b) the work done against the frictional force. [3]

(iii) Show that $21v^2 = 220x$. [2]

Answer: Kinetic Energy = $10.5\ v^2$ **J14/42/Q5**

Answer: Loss of Gravitational Potential Energy = $135x$

Answer: Work done against friction = $25x$

Answer: $135x = 10.5v^2 + 25x$ leading to the given expression

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45 One end of a light inextensible string is attached to a block. The string makes an angle of 60° above the horizontal and is used to pull the block in a straight line on a horizontal floor with acceleration $0.5\ \mathrm{m\,s^{-2}}$. The tension in the string is 8 N. The block starts to move with speed $0.3\ \mathrm{m\,s^{-1}}$. For the first 5 s of the block's motion, find

(i) the distance travelled, [2]

(ii) the work done by the tension in the string. [2]

Answer: **(i)** Distance travelled = 7.75 m **(ii)** Work done by the tension in the string = 31 J **J15/42/Q1**

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46 The total mass of a cyclist and his cycle is 80 kg. The resistance to motion is zero.

(i) The cyclist moves along a horizontal straight road working at a constant rate of P W. Find the value of P given that the cyclist's speed is $5\ \mathrm{m\,s^{-1}}$ when his acceleration is $1.2\ \mathrm{m\,s^{-2}}$. [2]

(ii) The cyclist moves up a straight hill inclined at an angle α, where $\sin\alpha = 0.035$. Find the acceleration of the cyclist at an instant when he is working at a rate of 450 W and has speed $3.6\ \mathrm{m\,s^{-1}}$. [3]

Answer: **(i)** $P = 480$ **(ii)** The acceleration of the cyclist is $\frac{97}{80} = 1.21 \text{ ms}^{-2}$ **J15/42/Q2**

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47 A plane is inclined at an angle of $\sin^{-1}(\frac{1}{8})$ to the horizontal. A and B are two points on the same line of greatest slope with A higher than B. The distance AB is 12 m. A small object P of mass 8 kg is released from rest at A and slides down the plane, passing through B with speed 4.5 m s^{-1}. For the motion of P from A to B, find

(i) the increase in kinetic energy of P and the decrease in potential energy of P, [3]

(ii) the magnitude of the constant resisting force that opposes the motion of P. [2]

Answer: **(i)** Increase in kinetic energy of P = 81 J Decrease in potential energy of P = 120 J
(ii) The magnitude of the constant resisting force is 3.25 N **J15/42/Q3**

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48 A racing cyclist, whose mass with his cycle is 75 kg, works at a rate of 720 W while moving on a straight horizontal road. The resistance to the cyclist's motion is constant and equal to R N.

(i) Given that the cyclist is accelerating at 0.16 m s^{-2} at an instant when his speed is 12 m s^{-1}, find the value of R. [3]

(ii) Given that the cyclist's acceleration is positive, show that his speed is less than 15 m s^{-1}. [2]

Answers: **(i)** $R = 48$ N; **(ii)** $v < 15 \text{ ms}^{-1}$ **N11/42/Q1**

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49 A lorry of mass 16 000 kg climbs a straight hill $ABCD$ which makes an angle θ with the horizontal, where $\sin\theta = \frac{1}{20}$. For the motion from A to B, the work done by the driving force of the lorry is 1200 kJ and the resistance to motion is constant and equal to 1240 N. The speed of the lorry is 15 m s^{-1} at A and 12 m s^{-1} at B.

(i) Find the distance AB. [5]

For the motion from B to D the gain in potential energy of the lorry is 2400 kJ.

(ii) Find the distance BD. [1]

For the motion from B to D the driving force of the lorry is constant and equal to 7200 N. From B to C the resistance to motion is constant and equal to 1240 N and from C to D the resistance to motion is constant and equal to 1860 N.

(iii) Given that the speed of the lorry at D is 7 m s^{-1}, find the distance BC. [4]

Answers: **(i)** 200m; **(ii)** 300m; **(iii)** 61.3 m **N11/42/Q6**

50

A block is pushed along a horizontal floor by a force of magnitude 45 N acting at an angle of 14° to the horizontal (see diagram). Find the work done by the force in moving the block a distance of 25 m. [3]

Answer: 1090 J. **N12/42/Q1**

51 A car of mass 1250 kg moves from the bottom to the top of a straight hill of length 500 m. The top of the hill is 30 m above the level of the bottom. The power of the car's engine is constant and equal to 30 000 W. The car's acceleration is 4 m s^{-2} at the bottom of the hill and is 0.2 m s^{-2} at the top. The resistance to the car's motion is 1000 N. Find

(i) the car's gain in kinetic energy, [5]

(ii) the work done by the car's engine. [3]

Answers: (i) 128 kJ; (ii) 1000 kJ. **N12/42/Q6**

52 A box of mass 25 kg is pulled in a straight line along a horizontal floor. The box starts from rest at a point A and has a speed of 3 m s^{-1} when it reaches a point B. The distance AB is 15 m. The pulling force has magnitude 220 N and acts at an angle of $\alpha°$ above the horizontal. The work done against the resistance to motion acting on the box, as the box moves from A to B, is 3000 J. Find the value of α. [5]

`2

53 The resistance to motion acting on a runner of mass 70 kg is kv N, where v m s^{-1} is the runner's speed and k is a constant. The greatest power the runner can exert is 100 W. The runner's greatest steady speed on horizontal ground is 4 m s^{-1}.

(i) Show that $k = 6.25$. [2]

(ii) Find the greatest steady speed of the runner while running uphill on a straight path inclined at an angle α to the horizontal, where $\sin\alpha = 0.05$. [4]

Answer: $v = 2.08$ ms^{-1} **N13/42/Q3**

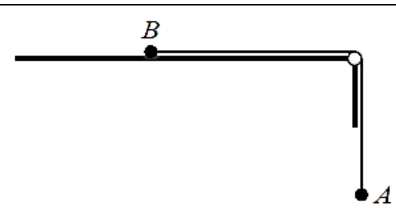
54 A train of mass 200 000 kg moves on a horizontal straight track. It passes through a point A with speed 28 m s^{-1} and later it passes through a point B. The power of the train's engine at B is 1.2 times the power of the train's engine at A. The driving force of the train's engine at B is 0.96 times the driving force of the train's engine at A.

(i) Show that the speed of the train at B is 35 m s^{-1}. [2]

(ii) For the motion from A to B, find the work done by the train's engine given that the work done against the resistance to the train's motion is 2.3×10^6 J. [3]

Answer: 4.64×10^6 **N14/42/Q3**

55



Particles A and B, each of mass 0.3 kg, are connected by a light inextensible string. The string passes over a small smooth pulley fixed at the edge of a rough horizontal surface. Particle A hangs freely and particle B is held at rest in contact with the surface (see diagram). The coefficient of friction between B and the surface is 0.7. Particle B is released and moves on the surface without reaching the pulley.

(i) Find, for the first 0.9 m of B's motion,

(a) the work done against the frictional force acting on B, [2]

(b) the loss of potential energy of the system, [1]

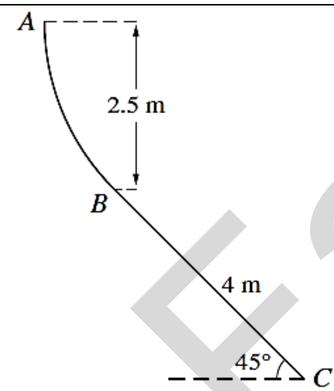
(c) the gain in kinetic energy of the system. [2]

At the instant when B has moved 0.9 m the string breaks. A is at a height of 0.54 m above a horizontal floor at this instant.

(ii) Find the speed with which A reaches the floor. [3]

Answer: 1.89, 2.7, 0.81, 3.67 **N14/42/Q5**

56



The diagram shows a vertical cross-section ABC of a surface. The part of the surface containing AB is smooth and A is 2.5 m above the level of B. The part of the surface containing BC is rough and is at 45° to the horizontal. The distance BC is 4 m (see diagram). A particle P of mass 0.2 kg is released from rest at A and moves in contact with the curve AB and then with the straight line BC. The coefficient of friction between P and the part of the surface containing BC is 0.4. Find the speed with which P reaches C. [6]

Answer: The speed with which P reaches C is 9.16 ms^{-1} **N15/42/Q4**

57 A car of mass 1600 kg moves with constant power 14 kW as it travels along a straight horizontal road. The car takes 25 s to travel between two points A and B on the road.

(i) Find the work done by the car's engine while the car travels from A to B. [2]

The resistance to the car's motion is constant and equal to 235 N. The car has accelerations at A and B of 0.5 $m\,s^{-2}$ and 0.25 $m\,s^{-2}$ respectively. Find

(ii) the gain in kinetic energy by the car in moving from A to B, [5]

(iii) the distance AB. [3]

Answer: Work Done by the car's engine = 350000 J = 350 kJ **N15/42/Q7**

Answer: V_A = 2800/207 = 13.53 ms^{-1} and V_B = 2800/127 = 22.05 ms^{-1} Gain in Kinetic Energy = 242490 J

Answer: The distance AB = 457 m

HOMEWORK: WORK, POWER, ENERGY VARIANT 41 & 43

1 A block is pulled along a horizontal floor by a horizontal rope. The tension in the rope is 500 N and the block moves at a constant speed of $2.75\ \text{m s}^{-1}$. Find the work done by the tension in 40 s and find the power applied by the tension. [4]

Answer: Work Done = 55000 J Power = 1375 W — 43/J15/1

2 A car of mass 860 kg travels along a straight horizontal road. The power provided by the car's engine is P W and the resistance to the car's motion is R N. The car passes through one point with speed $4.5\ \text{m s}^{-1}$ and acceleration $4\ \text{m s}^{-2}$. The car passes through another point with speed $22.5\ \text{m s}^{-1}$ and acceleration $0.3\ \text{m s}^{-2}$. Find the values of P and R. [6]

Answer: P = 17900 R = 537.5 — 43/J15/3

3 A lorry of mass 12 000 kg moves up a straight hill of length 500 m, starting at the bottom with a speed of $24\ \text{m s}^{-1}$ and reaching the top with a speed of $16\ \text{m s}^{-1}$. The top of the hill is 25 m above the level of the bottom of the hill. The resistance to motion of the lorry is 7500 N. Find the driving force of the lorry. [6]

Answer: Driving Force = 9660 N — 43/J15/4

4 A block of weight 6.1 N slides down a slope inclined at $\tan^{-1}\left(\frac{11}{60}\right)$ to the horizontal. The coefficient of friction between the block and the slope is $\frac{1}{4}$. The block passes through a point A with speed $2\ \text{m s}^{-1}$. Find how far the block moves from A before it comes to rest. [5]

Answer: Distance moved by the block from A before it comes to rest is 3.05 m — 41/J15/3

5 A lorry of mass 14 000 kg moves along a road starting from rest at a point O. It reaches a point A, and then continues to a point B which it reaches with a speed of $24\ \text{m s}^{-1}$. The part OA of the road is straight and horizontal and has length 400 m. The part AB of the road is straight and is inclined downwards at an angle of $\theta°$ to the horizontal and has length 300 m.

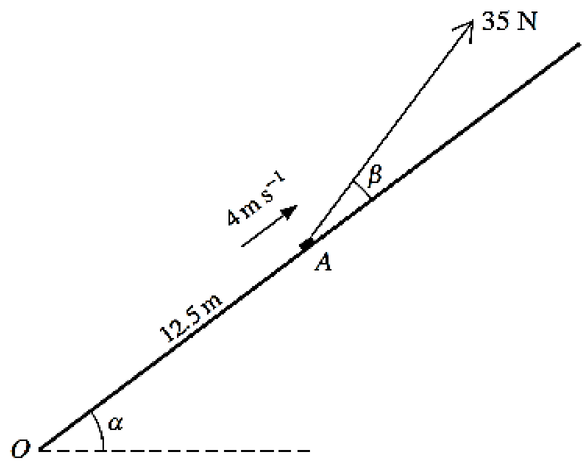
(i) For the motion from O to B, find the gain in kinetic energy of the lorry and express its loss in potential energy in terms of θ. [3]

The resistance to the motion of the lorry is 4800 N and the work done by the driving force of the lorry from O to B is 5000 kJ.

(ii) Find the value of θ. [3]

Answer: (i) Gain in Kinetic Energy = 4032000 J = 4032 kJ — 41/J15/4
Loss of Potential Energy = 42000000 sin θ J = 42000 sin θ kJ
(ii) θ = 3.3

6



A small block of mass 3 kg is initially at rest at the bottom O of a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.6$ and $\cos\alpha = 0.8$. A force of magnitude 35 N acts on the block at an angle β above the plane, where $\sin\beta = 0.28$ and $\cos\beta = 0.96$. The block starts to move up a line of greatest slope of the plane and passes through a point A with speed 4 m s^{-1}. The distance OA is 12.5 m (see diagram).

(i) For the motion of the block from O to A, find the work done against the frictional force acting on the block. [4]

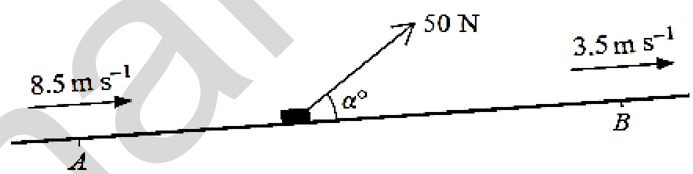
(ii) Find the coefficient of friction between the block and the plane. [3]

At the instant that the block passes through A the force of magnitude 35 N ceases to act.

(iii) Find the distance the block travels up the plane after passing through A. [4]

Answer: 171, 0.963, 0.584 — 43/N14/7

7



A block of mass 60 kg is pulled up a hill in the line of greatest slope by a force of magnitude 50 N acting at an angle $\alpha°$ above the hill. The block passes through points A and B with speeds 8.5 m s^{-1} and 3.5 m s^{-1} respectively (see diagram). The distance AB is 250 m and B is 17.5 m above the level of A. The resistance to motion of the block is 6 N. Find the value of α. [11]

Answer: 35.3 — 41/N14/7

8 A lorry of mass 16 000 kg travels at constant speed from the bottom, O, to the top, A, of a straight hill. The distance OA is 1200 m and A is 18 m above the level of O. The driving force of the lorry is constant and equal to 4500 N.

(i) Find the work done against the resistance to the motion of the lorry. [3]

On reaching A the lorry continues along a straight horizontal road against a constant resistance of 2000 N. The driving force of the lorry is not now constant, and the speed of the lorry increases from 9 m s^{-1} at A to 21 m s^{-1} at the point B on the road. The distance AB is 2400 m.

(ii) Use an energy method to find F, where F N is the average value of the driving force of the lorry while moving from A to B. [3]

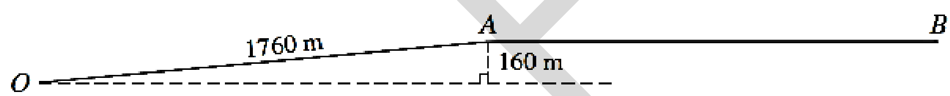
(iii) Given that the driving force at A is 1280 N greater than F N and that the driving force at B is 1280 N less than F N, show that the power developed by the lorry's engine is the same at B as it is at A. [2]

Answer: 2.52×10^6 J 3200 43/J14/5

9 A train is moving at constant speed V m s^{-1} along a horizontal straight track. Given that the power of the train's engine is 1330 kW and the total resistance to the train's motion is 28 kN, find the value of V. [3]

Answer: $V = 47.5$ 41/J14/1

10



A car of mass 1100 kg starts from rest at O and travels along a road OAB. The section OA is straight, of length 1760 m, and inclined to the horizontal with A at a height of 160 m above the level of O. The section AB is straight and horizontal (see diagram). While the car is moving the driving force of the car is 1800 N and the resistance to the car's motion is 700 N. The speed of the car is v m s^{-1} when the car has travelled a distance of x m from O.

(i) For the car's motion from O to A, write down its increase in kinetic energy in terms of v and its increase in potential energy in terms of x. Hence find the value of k for which $kv^2 = x$ for $0 \leqslant x \leqslant 1760$. [4]

(ii) Show that $v^2 = 2x - 3200$ for $x \geqslant 1760$. [4]

Answer: $k = 5.5$ 41/J14/5

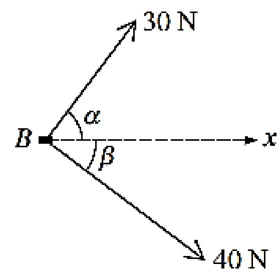
11 A lorry of mass 12 500 kg travels along a road from A to C passing through a point B. The resistance to motion of the lorry is 4800 N for the whole journey from A to C.

(i) The section AB of the road is straight and horizontal. On this section of the road the power of the lorry's engine is constant and equal to 144 kW. The speed of the lorry at A is 16 m s^{-1} and its acceleration at B is 0.096 m s^{-2}. Find the acceleration of the lorry at A and show that its speed at B is 24 m s^{-1}. [3]

(ii) The section BC of the road has length 500 m, is straight and inclined upwards towards C. On this section of the road the lorry's driving force is constant and equal to 5800 N. The speed of the lorry at C is 16 m s^{-1}. Find the height of C above the level of AB. [5]

Answer: 0.336 ms^{-2} 20 m 43/N13/6

12



A block B lies on a rough horizontal plane. Horizontal forces of magnitudes 30 N and 40 N, making angles of α and β respectively with the x-direction, act on B as shown in the diagram, and B is moving in the x-direction with constant speed. It is given that $\cos\alpha = 0.6$ and $\cos\beta = 0.8$.

(i) Find the total work done by the forces shown in the diagram when B has moved a distance of 20 m. [2]

(ii) Given that the coefficient of friction between the block and the plane is $\frac{5}{8}$, find the weight of the block. [3]

Answer: (i) 1000 J (ii) 80 N 41/N13/2

13 A lorry of mass 15 000 kg climbs from the bottom to the top of a straight hill, of length 1440 m, at a constant speed of $15\,\mathrm{m\,s^{-1}}$. The top of the hill is 16 m above the level of the bottom of the hill. The resistance to motion is constant and equal to 1800 N.

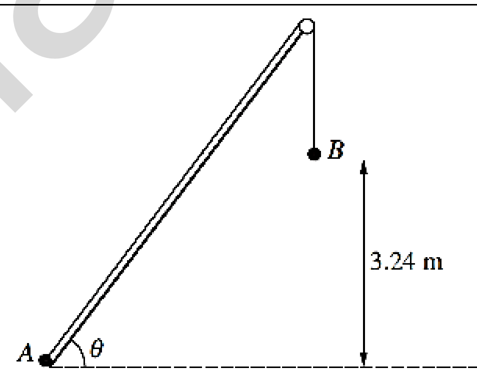
(i) Find the work done by the driving force. [4]

On reaching the top of the hill the lorry continues on a straight horizontal road and passes through a point P with speed $24\,\mathrm{m\,s^{-1}}$. The resistance to motion is constant and is now equal to 1600 N. The work done by the lorry's engine from the top of the hill to the point P is 5030 kJ.

(ii) Find the distance from the top of the hill to the point P. [3]

Answer: (i) 4.99×10^6 J (ii) 1500 m 41/N13/5

14



Particle A of mass 1.6 kg and particle B of mass 2 kg are attached to opposite ends of a light inextensible string. The string passes over a small smooth pulley fixed at the top of a smooth plane, which is inclined at angle θ, where $\sin\theta = 0.8$. Particle A is held at rest at the bottom of the plane and B hangs at a height of 3.24 m above the level of the bottom of the plane (see diagram). A is released from rest and the particles start to move.

(i) Show that the loss of potential energy of the system, when B reaches the level of the bottom of the plane, is 23.328 J. [3]

(ii) Hence find the speed of the particles when B reaches the level of the bottom of the plane. [2]

Answer. 23.328 J; 3.6 ms^{-2} — 43/J13/2

15 A car has mass 800 kg. The engine of the car generates constant power P kW as the car moves along a straight horizontal road. The resistance to motion is constant and equal to R N. When the car's speed is 14 m s^{-1} its acceleration is 1.4 m s^{-2}, and when the car's speed is 25 m s^{-1} its acceleration is 0.33 m s^{-2}. Find the values of P and R. [6]

Answer. 27.2; 825 — 43/J13/3

16 A car of mass 1250 kg travels from the bottom to the top of a straight hill of length 600 m, which is inclined at an angle of 2.5° to the horizontal. The resistance to motion of the car is constant and equal to 400 N. The work done by the driving force is 450 kJ. The speed of the car at the bottom of the hill is 30 m s^{-1}. Find the speed of the car at the top of the hill. [5]

Answer: 26.7 — 41/J13/2

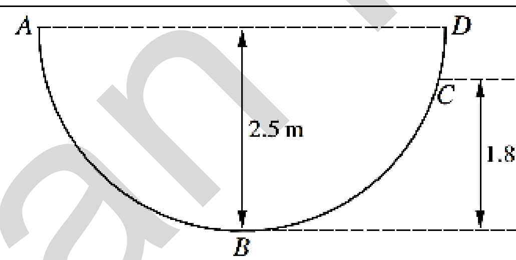
17 A train of mass 400 000 kg is moving on a straight horizontal track. The power of the engine is constant and equal to 1500 kW and the resistance to the train's motion is 30 000 N. Find

(i) the acceleration of the train when its speed is 37.5 m s^{-1}, [4]

(ii) the steady speed at which the train can move. [2]

Answer: i) 0.025 ii) 50 — 41/J13/4

18



$ABCD$ is a semi-circular cross-section, in a vertical plane, of the inner surface of half a hollow cylinder of radius 2.5 m which is fixed with its axis horizontal. AD is horizontal, B is the lowest point of the cross-section and C is at a height of 1.8 m above the level of B (see diagram). A particle P of mass 0.8 kg is released from rest at A and comes to instantaneous rest at C.

(i) Find the work done on P by the resistance to motion while P travels from A to C. [2]

The work done on P by the resistance to motion while P travels from A to B is 0.6 times the work done while P travels from A to C.

(ii) Find the speed of P when it passes through B. [3]

Answers. (i) 5.6 J; (ii) 6.45 ms^{-1}. — 43/N12/1

19 An object of mass 12 kg slides down a line of greatest slope of a smooth plane inclined at 10° to the horizontal. The object passes through points *A* and *B* with speeds $3\ \text{m s}^{-1}$ and $7\ \text{m s}^{-1}$ respectively.

(i) Find the increase in kinetic energy of the object as it moves from *A* to *B*. [2]

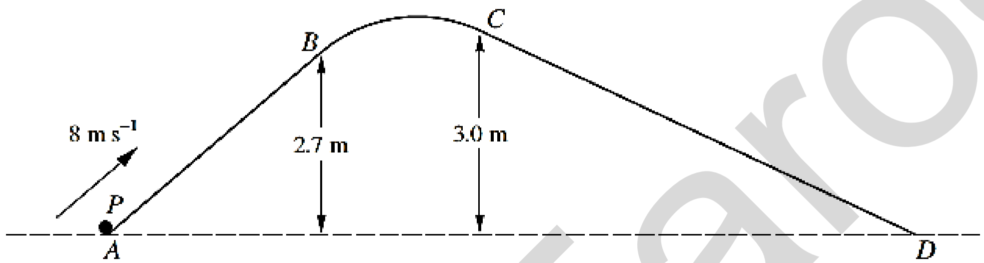
(ii) Hence find the distance *AB*, assuming there is no resisting force acting on the object. [3]

The object is now pushed up the plane from *B* to *A*, with constant speed, by a horizontal force.

(iii) Find the magnitude of this force. [3]

Answers: (i) 240 J; (ii) 11.5 m; (iii) 21.2 N. 43/N12/5

20



The diagram shows the vertical cross-section *ABCD* of a surface. *BC* is a circular arc, and *AB* and *CD* are tangents to *BC* at *B* and *C* respectively. *A* and *D* are at the same horizontal level, and *B* and *C* are at heights 2.7 m and 3.0 m respectively above the level of *A* and *D*. A particle *P* of mass 0.2 kg is given a velocity of $8\ \text{m s}^{-1}$ at *A*, in the direction of *AB* (see diagram). The parts of the surface containing *AB* and *BC* are smooth.

(i) Find the decrease in the speed of *P* as *P* moves along the surface from *B* to *C*. [4]

The part of the surface containing *CD* exerts a constant frictional force on *P*, as it moves from *C* to *D*, and *P* comes to rest as it reaches *D*.

(ii) Find the speed of *P* when it is at the mid-point of *CD*. [5]

Answers: (i) $1.16\ \text{ms}^{-1}$; (ii) $1.41\ \text{ms}^{-1}$. 41/N12/6

21 A car of mass 1200 kg moves in a straight line along horizontal ground. The resistance to motion of the car is constant and has magnitude 960 N. The car's engine works at a rate of 17 280 W.

(i) Calculate the acceleration of the car at an instant when its speed is $12\ \text{m s}^{-1}$. [3]

The car passes through the points *A* and *B*. While the car is moving between *A* and *B* it has constant speed $V\ \text{m s}^{-1}$.

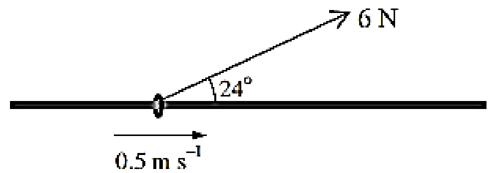
(ii) Show that $V = 18$. [2]

At the instant that the car reaches *B* the engine is switched off and subsequently provides no energy. The car continues along the straight line until it comes to rest at the point *C*. The time taken for the car to travel from *A* to *C* is 52.5 s.

(iii) Find the distance *AC*. [5]

Answers: (i) $0.4\ \text{ms}^{-2}$; (ii) 742.5 m. 41/N12/7

22



A ring is threaded on a fixed horizontal bar. The ring is attached to one end of a light inextensible string which is used to pull the ring along the bar at a constant speed of $0.5\ \text{m s}^{-1}$. The string makes a constant angle of 24° with the bar and the tension in the string is 6 N (see diagram). Find the work done by the tension in a period of 8 s. [3]

Answer: 21.9 J 43/J12/1

23 A car of mass 1230 kg increases its speed from $4\ \text{m s}^{-1}$ to $21\ \text{m s}^{-1}$ in 24.5 s. The table below shows corresponding values of time t s and speed $v\ \text{m s}^{-1}$.

t	0	0.5	16.3	24.5
v	4	6	19	21

(i) Using the values in the table, find the average acceleration of the car for $0 < t < 0.5$ and for $16.3 < t < 24.5$. [2]

While the car is increasing its speed the power output of its engine is constant and equal to P W, and the resistance to the car's motion is constant and equal to R N.

(ii) Assuming that the values obtained in part (i) are approximately equal to the accelerations at $v = 5$ and at $v = 20$, find approximations for P and R. [5]

Answers: P=30800 W R=1240 N 43/J12/4

24 A lorry of mass 16 000 kg moves on a straight hill inclined at angle $\alpha°$ to the horizontal. The length of the hill is 500 m.

(i) While the lorry moves from the bottom to the top of the hill at constant speed, the resisting force acting on the lorry is 800 N and the work done by the driving force is 2800 kJ. Find the value of α. [4]

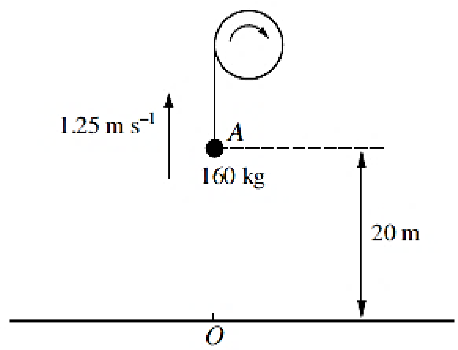
(ii) On the return journey the speed of the lorry is $20\ \text{m s}^{-1}$ at the top of the hill. While the lorry travels down the hill, the work done by the driving force is 2400 kJ and the work done against the resistance to motion is 800 kJ. Find the speed of the lorry at the bottom of the hill. [4]

Answers: (i) $\alpha = 1.7$ (ii) $v = 30\ \text{ms}^{-1}$ 43/J12/5

25 A car of mass 880 kg travels along a straight horizontal road with its engine working at a constant rate of P W. The resistance to motion is 700 N. At an instant when the car's speed is $16\ \text{m s}^{-1}$ its acceleration is $0.625\ \text{m s}^{-2}$. Find the value of P. [4]

Answer: 20 000 W 41/J12/1

26



A load of mass 160 kg is pulled vertically upwards, from rest at a fixed point O on the ground, using a winding drum. The load passes through a point A, 20 m above O, with a speed of 1.25 m s^{-1} (see diagram). Find, for the motion from O to A,

(i) the gain in the potential energy of the load, [1]

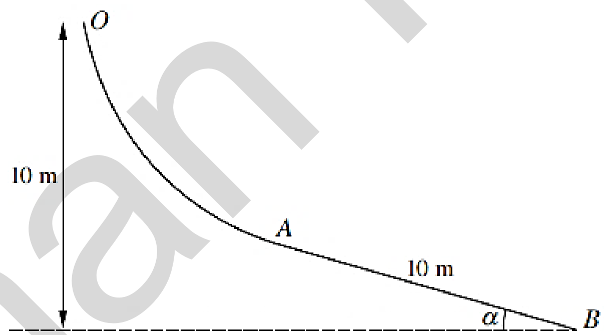
(ii) the gain in the kinetic energy of the load. [2]

The power output of the winding drum is constant while the load is in motion.

(iii) Given that the work done against the resistance to motion from O to A is 20 kJ and that the time taken for the load to travel from O to A is 41.7 s, find the power output of the winding drum. [3]

Answers: (i) 32 000 J (ii) 125 J (iii) 1250 W 41/J12/3

27



The diagram shows the vertical cross-section OAB of a slide. The straight line AB is tangential to the curve OA at A. The line AB is inclined at α to the horizontal, where $\sin\alpha = 0.28$. The point O is 10 m higher than B, and AB has length 10 m (see diagram). The part of the slide containing the curve OA is smooth and the part containing AB is rough. A particle P of mass 2 kg is released from rest at O and moves down the slide.

(i) Find the speed of P when it passes through A. [3]

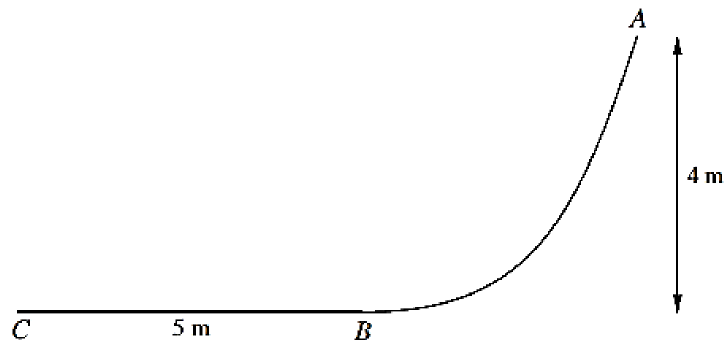
The coefficient of friction between P and the part of the slide containing AB is $\frac{1}{12}$. Find

(ii) the acceleration of P when it is moving from A to B, [3]

(iii) the speed of P when it reaches B. [2]

Answers: (i) 12 ms^{-1} (ii) 2 ms^{-2} (iii) 13.6 ms^{-1} 41/J12/5

28



ABC is a vertical cross-section of a surface. The part of the surface containing AB is smooth and A is 4 m higher than B. The part of the surface containing BC is horizontal and the distance BC is 5 m (see diagram). A particle of mass 0.8 kg is released from rest at A and slides along ABC. Find the speed of the particle at C in each of the following cases.

(i) The horizontal part of the surface is smooth. [3]

(ii) The coefficient of friction between the particle and the horizontal part of the surface is 0.3. [3]

Answers: (i) 8.94 ms^{-2}; (ii) 7.07 ms^{-2} 43/N11/4

29 A car of mass 600 kg travels along a straight horizontal road starting from a point A. The resistance to motion of the car is 750 N.

(i) The car travels from A to B at constant speed in 100 s. The power supplied by the car's engine is constant and equal to 30 kW. Find the distance AB. [3]

(ii) The car's engine is switched off at B and the car's speed decreases until the car reaches C with a speed of $20\,m\,s^{-1}$. Find the distance BC. [3]

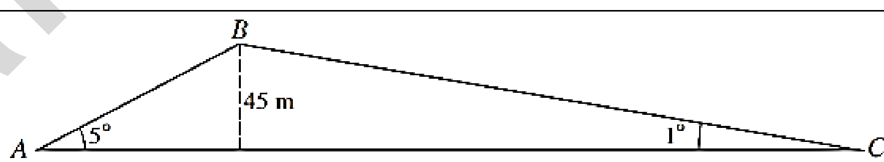
(iii) The car's engine is switched on at C and the power it supplies is constant and equal to 30 kW. The car takes 14 s to travel from C to D and reaches D with a speed of $30\,m\,s^{-1}$. Find the distance CD. [4]

Answers: (i) 4000 m; (ii) 480 m; (iii) 360 m 43/N11/7

30 One end of a light inextensible string is attached to a block. The string is used to pull the block along a horizontal surface with a speed of $2\,m\,s^{-1}$. The string makes an angle of 20° with the horizontal and the tension in the string is 25 N. Find the work done by the tension in a period of 8 seconds. [3]

Answer: 376 J 41/N11/1

31



AB and BC are straight roads inclined at 5° to the horizontal and 1° to the horizontal respectively. A and C are at the same horizontal level and B is 45 m above the level of A and C (see diagram, which is not to scale). A car of mass 1200 kg travels from A to C passing through B.

(i) For the motion from A to B, the speed of the car is constant and the work done against the resistance to motion is 360 kJ. Find the work done by the car's engine from A to B. [3]

The resistance to motion is constant throughout the whole journey.

(ii) For the motion from B to C the work done by the driving force is 1660 kJ. Given that the speed of the car at B is 15 m s^{-1}, show that its speed at C is 29.9 m s^{-1}, correct to 3 significant figures. [4]

(iii) The car's driving force immediately after leaving B is 1.5 times the driving force immediately before reaching C. Find, correct to 2 significant figures, the ratio of the power developed by the car's engine immediately after leaving B to the power developed immediately before reaching C. [3]

Answers: **(i)** Work done is 900 000 J or 900 kJ; **(iii)** Ratio is 0.75 — 41/N11/6

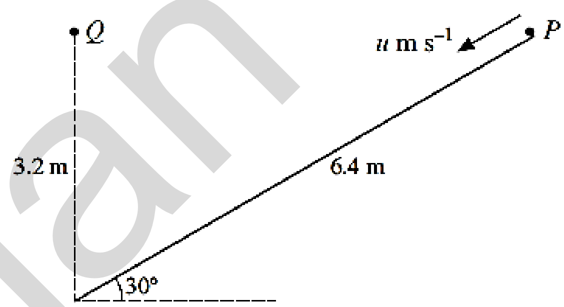
32 A block is pulled for a distance of 50 m along a horizontal floor, by a rope that is inclined at an angle of $\alpha°$ to the floor. The tension in the rope is 180 N and the work done by the tension is 8200 J. Find the value of α. [3]

Answer: 24.3. — 43/J11/1

33 A car of mass 1250 kg is travelling along a straight horizontal road with its engine working at a constant rate of P W. The resistance to the car's motion is constant and equal to R N. When the speed of the car is 19 m s^{-1} its acceleration is 0.6 m s^{-2}, and when the speed of the car is 30 m s^{-1} its acceleration is 0.16 m s^{-2}. Find the values of P and R. [6]

Answers: 28500; 750. — 43/J11/2

34



A particle P is projected from the top of a smooth ramp with speed u m s^{-1}, and travels down a line of greatest slope. The ramp has length 6.4 m and is inclined at 30° to the horizontal. Another particle Q is released from rest at a point 3.2 m vertically above the bottom of the ramp, at the same instant that P is projected (see diagram). Given that P and Q reach the bottom of the ramp simultaneously, find

(i) the value of u, [4]

(ii) the speed with which P reaches the bottom of the ramp. [2]

Answers: **(i)** 6; **(ii)** 10 ms^{-1}. — 43/J11/3

35 A lorry of mass 15 000 kg climbs a hill of length 500 m at a constant speed. The hill is inclined at 2.5° to the horizontal. The resistance to the lorry's motion is constant and equal to 800 N.

(i) Find the work done by the lorry's driving force. [4]

On its return journey the lorry reaches the top of the hill with speed 20 m s^{-1} and continues down the hill with a constant driving force of 2000 N. The resistance to the lorry's motion is again constant and equal to 800 N.

(ii) Find the speed of the lorry when it reaches the bottom of the hill. [5]

Answers: **(i)** 3 670 000 J; **(ii)** 30.3 ms^{-1}. 43/J11/6

36 A car of mass 700 kg is travelling along a straight horizontal road. The resistance to motion is constant and equal to 600 N.

(i) Find the driving force of the car's engine at an instant when the acceleration is 2 m s^{-2}. [2]

(ii) Given that the car's speed at this instant is 15 m s^{-1}, find the rate at which the car's engine is working. [2]

Answers: **(i)** 2000 N; **(ii)** 30 000 W (or 30 kW). 41/J11/1

37 A load of mass 1250 kg is raised by a crane from rest on horizontal ground, to rest at a height of 1.54 m above the ground. The work done against the resistance to motion is 5750 J.

(i) Find the work done by the crane. [3]

(ii) Assuming the power output of the crane is constant and equal to 1.25 kW, find the time taken to raise the load. [2]

Answers: **(i)** 25 000 J (or 25 kJ); **(ii)** 20 s. 41/J11/2

38 Loads A and B, of masses 1.2 kg and 2.0 kg respectively, are attached to the ends of a light inextensible string which passes over a fixed smooth pulley. A is held at rest and B hangs freely, with both straight parts of the string vertical. A is released and starts to move upwards. It does not reach the pulley in the subsequent motion.

(i) Find the acceleration of A and the tension in the string. [4]

(ii) Find, for the first 1.5 metres of A's motion,

(a) A's gain in potential energy,

(b) the work done on A by the tension in the string,

(c) A's gain in kinetic energy.

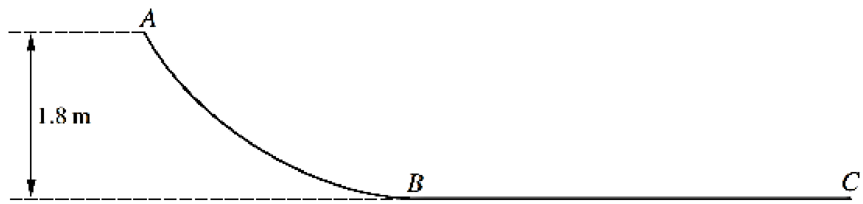
[3]

B hits the floor 1.6 seconds after A is released. B comes to rest without rebounding and the string becomes slack.

(iii) Find the time from the instant the string becomes slack until it becomes taut again. [4]

Answers: **(i)** 2.5 ms^{-2}; 15 N; **(ii)(a)** 18 J, **(b)** 22.5 J, **(c)** 4.5 J; **(iii)** 0.8 s. 41/J11/7

39



The diagram shows the vertical cross-section ABC of a fixed surface. AB is a curve and BC is a horizontal straight line. The part of the surface containing AB is smooth and the part containing BC is rough. A is at a height of 1.8 m above BC. A particle of mass 0.5 kg is released from rest at A and travels along the surface to C.

(i) Find the speed of the particle at B. [2]

(ii) Given that the particle reaches C with a speed of 5 m s^{-1}, find the work done against the resistance to motion as the particle moves from B to C. [2]

Answers: (i) 6 ms^{-1}; (ii) 2.75 J. 43/N10/2

40 A car of mass 1250 kg travels along a horizontal straight road. The power of the car's engine is constant and equal to 24 kW and the resistance to the car's motion is constant and equal to R N. The car passes through the point A on the road with speed 20 m s^{-1} and acceleration 0.32 m s^{-2}.

(i) Find the value of R. [3]

The car continues with increasing speed, passing through the point B on the road with speed 29.9 m s^{-1}. The car subsequently passes through the point C.

(ii) Find the acceleration of the car at B, giving the answer in m s^{-2} correct to 3 decimal places. [2]

(iii) Show that, while the car's speed is increasing, it cannot reach 30 m s^{-1}. [2]

(iv) Explain why the speed of the car is approximately constant between B and C. [1]

(v) State a value of the approximately constant speed, and the maximum possible error in this value at any point between B and C. [1]

The work done by the car's engine during the motion from B to C is 1200 kJ.

(vi) By assuming the speed of the car is constant from B to C, find, in either order,

(a) the approximate time taken for the car to travel from B to C,

(b) an approximation for the distance BC.

[4]

Answers: (i) 800; (ii) 0.002 ms^{-2}; (v) 30 ms^{-1}, 0.1 ms^{-1}; (vi)(a) 50 s, (b) 1500 m. 43/N10/7

41 A car of mass 600 kg travels along a horizontal straight road, with its engine working at a rate of 40 kW. The resistance to motion of the car is constant and equal to 800 N. The car passes through the point A on the road with speed 25 m s^{-1}. The car's acceleration at the point B on the road is half its acceleration at A. Find the speed of the car at B. [5]

Answer: 33.3 41/N10/2

42 A particle of mass 0.8 kg slides down a rough inclined plane along a line of greatest slope AB. The distance AB is 8 m. The particle starts at A with speed 3 m s^{-1} and moves with constant acceleration 2.5 m s^{-2}.

(i) Find the speed of the particle at the instant it reaches B. [2]

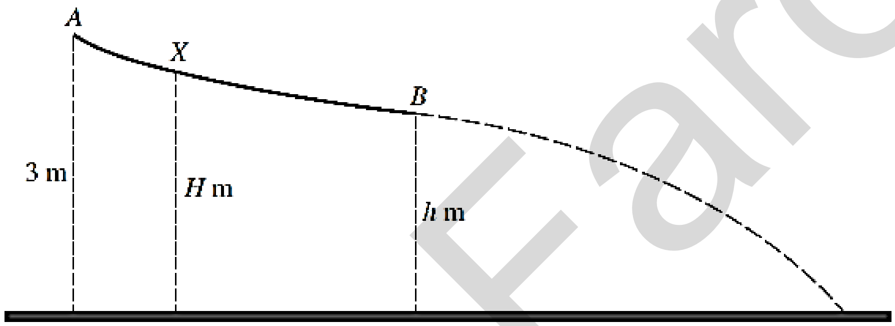
(ii) Given that the work done against the frictional force as the particle moves from A to B is 7 J, find the angle of inclination of the plane. [4]

When the particle is at the point X its speed is the same as the average speed for the motion from A to B.

(iii) Find the work done by the frictional force for the particle's motion from A to X. [3]

Answer: i) 7 ii) 21.1 degrees iii) 2.8 — 41/N10/5

43



A smooth slide AB is fixed so that its highest point A is 3 m above horizontal ground. B is h m above the ground. A particle P of mass 0.2 kg is released from rest at a point on the slide. The particle moves down the slide and, after passing B, continues moving until it hits the ground (see diagram). The speed of P at B is v_B and the speed at which P hits the ground is v_G.

(i) In the case that P is released at A, it is given that the kinetic energy of P at B is 1.6 J. Find

(a) the value of h, [3]

(b) the kinetic energy of the particle immediately before it reaches the ground, [1]

(c) the ratio $v_G : v_B$. [2]

(ii) In the case that P is released at the point X of the slide, which is H m above the ground (see diagram), it is given that $v_G : v_B = 2.55$. Find the value of H correct to 2 significant figures. [3]

Answer: i) a) 2.2 b) 6 c) 1.94 ii) 2.6 — 41/N10/6

44 A load is pulled along a horizontal straight track, from A to B, by a force of magnitude P N which acts at an angle of 30° upwards from the horizontal. The distance AB is 80 m. The speed of the load is constant and equal to 1.2 m s^{-1} as it moves from A to the mid-point M of AB.

(i) For the motion from A to M the value of P is 25. Calculate the work done by the force as the load moves from A to M. [2]

The speed of the load increases from 1.2 m s^{-1} as it moves from M towards B. For the motion from M to B the value of P is 50 and the work done against resistance is the same as that for the motion from A to M. The mass of the load is 35 kg.

(ii) Find the gain in kinetic energy of the load as it moves from M to B and hence find the speed with which it reaches B. [5]

Answers: (i) 866 J; (ii) 866 J, 7.14 ms^{-1}. 43/J10/3

45 A car of mass 1150 kg travels up a straight hill inclined at 1.2° to the horizontal. The resistance to motion of the car is 975 N. Find the acceleration of the car at an instant when it is moving with speed 16 $m s^{-1}$ and the engine is working at a power of 35 kW. [4]

Answer: 0.845 41/J10/1

46 P and Q are fixed points on a line of greatest slope of an inclined plane. The point Q is at a height of 0.45 m above the level of P. A particle of mass 0.3 kg moves upwards along the line PQ.

(i) Given that the plane is smooth and that the particle just reaches Q, find the speed with which it passes through P. [3]

(ii) It is given instead that the plane is rough. The particle passes through P with the same speed as that found in part **(i)**, and just reaches a point R which is between P and Q. The work done against the frictional force in moving from P to R is 0.39 J. Find the potential energy gained by the particle in moving from P to R and hence find the height of R above the level of P. [4]

Answer: i) 3 ii) 0.32 41/J10/5

47 A box of mass 25 kg is pulled, at a constant speed, a distance of 36 m up a rough plane inclined at an angle of 20° to the horizontal. The box moves up a line of greatest slope against a constant frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope. Find

(i) the work done against friction, [1]

(ii) the change in gravitational potential energy of the box, [2]

(iii) the work done by the pulling force. [2]

Answers: Work done against friction = 1440 J *Answer*: Change in GPE = 3080 J J16/41/Q2

Answer: Work done by the pulling force is 4520 J

48 A car of mass 1000 kg is moving along a straight horizontal road against resistances of total magnitude 300 N.

(i) Find, in kW, the rate at which the engine of the car is working when the car has a constant speed of 40 $m s^{-1}$. [3]

(ii) Find the acceleration of the car when its speed is 25 $m s^{-1}$ and the engine is working at 90% of the power found in part **(i)**. [3]

Answer: Rate at which the engine is working is 12 kW J16/41/Q3

Answer: The acceleration of the car is 0.132 ms^{-2}

49 A particle of mass 30 kg is on a plane inclined at an angle of 20° to the horizontal. Starting from rest, the particle is pulled up the plane by a force of magnitude 200 N acting parallel to a line of greatest slope.

(i) Given that the plane is smooth, find

(a) the acceleration of the particle, [2]

(b) the change in kinetic energy after the particle has moved 12 m up the plane. [2]

(ii) It is given instead that the plane is rough and the coefficient of friction between the particle and the plane is 0.12.

(a) Find the acceleration of the particle. [4]

(b) The direction of the force of magnitude 200 N is changed, and the force now acts at an angle of 10° above the line of greatest slope. Find the acceleration of the particle. [4]

Answer: Acceleration of the particle = 3.25 ms^{-2} J16/41/Q7

Answer: Change in kinetic energy = 1170 J

Answer: The acceleration of the particle is 2.12 ms^{-2}

Answer: The acceleration of the particle is 2.16 ms^{-2}

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50 A particle of mass 8 kg is pulled at a constant speed a distance of 20 m up a rough plane inclined at an angle of 30° to the horizontal by a force acting along a line of greatest slope.

(i) Find the change in gravitational potential energy of the particle. [2]

(ii) The total work done against gravity and friction is 1146 J. Find the frictional force acting on the particle. [2]

Answer: 800 J 17.3 N J16/43/Q1

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51 The motion of a car of mass 1400 kg is resisted by a constant force of magnitude 650 N.

(i) Find the constant speed of the car on a horizontal road, assuming that the engine works at a rate of 20 kW. [2]

(ii) The car is travelling at a constant speed of $10\,m\,s^{-1}$ up a hill inclined at an angle of θ to the horizontal, where $\sin\theta = \frac{1}{7}$. Find the power of the car's engine. [3]

(iii) The car descends the same hill with the engine working at 80% of the power found in part **(ii)**. Find the acceleration of the car at an instant when the speed is $20\,m\,s^{-1}$. [3]

Answer: 30.8 ms^{-1} 26500 W 1.72 ms^{-2} J16/43/Q5

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52 A particle of mass 0.6 kg is dropped from a height of 8 m above the ground. The speed of the particle at the instant before hitting the ground is $10\,m\,s^{-1}$. Find the work done against air resistance. [3]

Answers: Work done against air resistance is 18 J J17/41/Q1

53 A car of mass 800 kg is moving up a hill inclined at $\theta°$ to the horizontal, where $\sin\theta = 0.15$. The initial speed of the car is $8\,\text{m s}^{-1}$. Twelve seconds later the car has travelled 120 m up the hill and has speed $14\,\text{m s}^{-1}$.

(i) Find the change in the kinetic energy and the change in gravitational potential energy of the car. [3]

(ii) The engine of the car is working at a constant rate of 32 kW. Find the total work done against the resistive forces during the twelve seconds. [3]

Answers: (i) Change in kinetic energy = 52 800 J Change in gravitational potential energy = 144 000 J
(ii) The total work done against the resistive forces during the 12 seconds is 187 200 J (187 000 to three significant figures) J17/41/Q4

54 A car of mass 1200 kg is travelling along a horizontal road.

(i) It is given that there is a constant resistance to motion.

(a) The engine of the car is working at 16 kW while the car is travelling at a constant speed of $40\,\text{m s}^{-1}$. Find the resistance to motion. [2]

(b) The power is now increased to 22.5 kW. Find the acceleration of the car at the instant it is travelling at a speed of $45\,\text{m s}^{-1}$. [3]

(ii) It is given instead that the resistance to motion of the car is $(590 + 2v)$ N when the speed of the car is $v\,\text{m s}^{-1}$. The car travels at a constant speed with the engine working at 16 kW. Find this speed. [3]

Answers: (i)(a) 400 N (b) 0.0833 ms^{-2} (ii) 25 ms^{-2} J17/43/Q6

55 A car has mass 1250 kg.

(i) The car is moving along a straight level road at a constant speed of $36\,\text{m s}^{-1}$ and is subject to a constant resistance of magnitude 850 N. Find, in kW, the rate at which the engine of the car is working. [2]

(ii) The car travels at a constant speed up a hill and is subject to the same resistance as in part **(i)**. The hill is inclined at an angle of $\theta°$ to the horizontal, where $\sin\theta° = 0.1$, and the engine is working at 63 kW. Find the speed of the car. [3]

(iii) The car descends the same hill with the engine of the car working at a constant rate of 20 kW. The resistance is not constant. The initial speed of the car is $20\,\text{m s}^{-1}$. Eight seconds later the car has speed $24\,\text{m s}^{-1}$ and has moved 176 m down the hill. Use an energy method to find the total work done against the resistance during the eight seconds. [5]

Answers: (i) The rate at which the engine of the car is working is 30.6 kW (ii) The speed of the car is 30 ms^{-1}
(iii) The work done against the resistance is 270 000 J J18/41/Q6

56 A car of mass 1400 kg travelling at a speed of $v\,\text{m}\,\text{s}^{-1}$ experiences a resistive force of magnitude $40v$ N. The greatest possible constant speed of the car along a straight level road is $56\,\text{m}\,\text{s}^{-1}$.

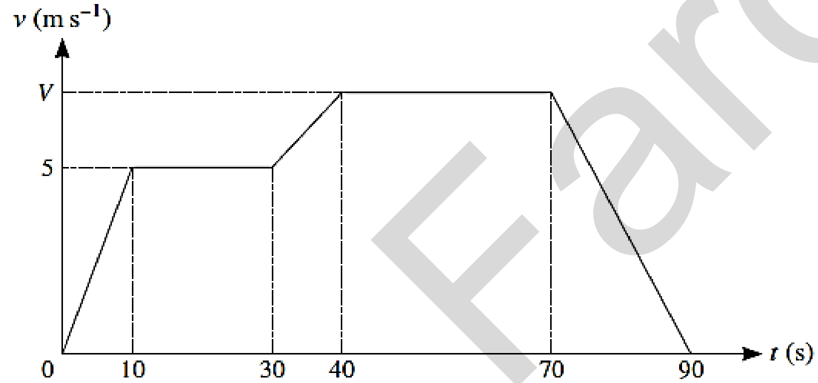
(i) Find, in kW, the greatest possible power of the car's engine. [2]

(ii) Find the greatest possible acceleration of the car at an instant when its speed on a straight level road is $32\,\text{m}\,\text{s}^{-1}$. [3]

(iii) The car travels down a hill inclined at an angle of $\theta°$ to the horizontal at a constant speed of $50\,\text{m}\,\text{s}^{-1}$. The power of the car's engine is 60 kW. Find the value of θ. [4]

Answers: (i) 125 kW (ii) $1.89\,\text{ms}^{-2}$ (iii) 3.3 J18/43/Q6

57



The diagram shows a velocity-time graph which models the motion of a cyclist. The graph consists of five straight line segments. The cyclist accelerates from rest to a speed of $5\,\text{m}\,\text{s}^{-1}$ over a period of 10 s, and then travels at this speed for a further 20 s. The cyclist then descends a hill, accelerating to speed $V\,\text{m}\,\text{s}^{-1}$ over a period of 10 s. This speed is maintained for a further 30 s. The cyclist then decelerates to rest over a period of 20 s.

(i) Find the acceleration of the cyclist during the first 10 seconds. [1]

(ii) Show that the total distance travelled by the cyclist in the 90 seconds of motion may be expressed as $(45V + 150)$ m. Hence find V, given that the total distance travelled by the cyclist is 465 m. [3]

(iii) The combined mass of the cyclist and the bicycle is 80 kg. The cyclist experiences a constant resistance to motion of 20 N. Use an energy method to find the vertical distance which the cyclist descends during the downhill section from $t = 30$ to $t = 40$, assuming that the cyclist does no work during this time. [4]

Answer: The acceleration of the cyclist over the first ten seconds is $0.5\,\text{ms}^{-2}$ N16/41/Q5

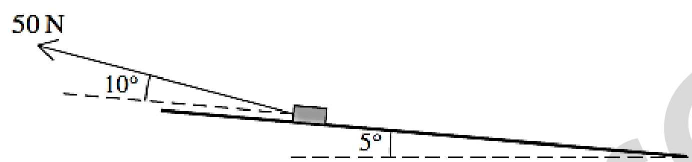
Answer: $V = 7$ *Answer*: The vertical distance which the cyclist descends is 2.7 m

58 A block of mass 25 kg is pulled along horizontal ground by a force of magnitude 50 N inclined at 10° above the horizontal. The block starts from rest and travels a distance of 20 m. There is a constant resistance force of magnitude 30 N opposing motion.

(i) Find the work done by the pulling force. [2]

(ii) Use an energy method to find the speed of the block when it has moved a distance of 20 m. [2]

(iii) Find the greatest power exerted by the 50 N force. [2]



After the block has travelled the 20 m, it comes to a plane inclined at 5° to the horizontal. The force of 50 N is now inclined at an angle of 10° to the plane and pulls the block directly up the plane (see diagram). The resistance force remains 30 N.

(iv) Find the time it takes for the block to come to rest from the instant when it reaches the foot of the inclined plane. [4]

Answer: Work done by the pulling force is 985 J

N16/41/Q6

Answer: The speed of the block after it has moved 20 m is 5.55 ms^{-1}

Answer: The greatest power exerted by the 50 N force is 273 W

Answer: t = 54.4 seconds

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59 A crane is used to raise a block of mass 50 kg vertically upwards at constant speed through a height of 3.5 m. There is a constant resistance to motion of 25 N.

(i) Find the work done by the crane. [3]

(ii) Given that the time taken to raise the block is 2 s, find the power of the crane. [2]

Answer: Work done by the crane is 1840 J *Answer:* Power of the crane is 919 W

N16/43/Q1

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60 A cyclist is cycling with constant power of 160 W along a horizontal straight road. There is a constant resistance to motion of 20 N. At an instant when the cyclist's speed is 5 $m s^{-1}$, his acceleration is 0.15 $m s^{-2}$.

(i) Show that the total mass of the cyclist and bicycle is 80 kg. [3]

The cyclist comes to a hill inclined at 2° to the horizontal. When the cyclist starts climbing the hill, he increases his power to a constant 300 W. The resistance to motion remains 20 N.

(ii) Show that the steady speed up the hill which the cyclist can maintain when working at this power is 6.26 $m s^{-1}$, correct to 3 significant figures. [2]

(iii) Find the acceleration at an instant when the cyclist is travelling at 90% of the speed in part (**ii**). [4]

Answer: Total mass of cyclist and bicycle is 80 kg

Answer: Acceleration of the cyclist is 0.0666 ms^{-2}

N16/43/Q6

61 A tractor of mass 3700 kg is travelling along a straight horizontal road at a constant speed of 12 $m\,s^{-1}$. The total resistance to motion is 1150 N.

(i) Find the power output of the tractor's engine. [1]

The tractor comes to a hill inclined at 4° above the horizontal. The power output is increased to 25 kW and the resistance to motion is unchanged.

(ii) Find the deceleration of the tractor at the instant it begins to climb the hill. [3]

(iii) Find the constant speed that the tractor could maintain on the hill when working at this power. [2]

N17/41/Q2

62 A roller-coaster car (including passengers) has a mass of 840 kg. The roller-coaster ride includes a section where the car climbs a straight ramp of length 8 m inclined at 30° above the horizontal. The car then immediately descends another ramp of length 10 m inclined at 20° below the horizontal. The resistance to motion acting on the car is 640 N throughout the motion.

(i) Find the total work done against the resistance force as the car ascends the first ramp and descends the second ramp. [2]

(ii) The speed of the car at the bottom of the first ramp is 14 $m\,s^{-1}$. Use an energy method to find the speed of the car when it reaches the bottom of the second ramp. [4]

Answer: The total work done against the resistance force is 11 520 J

Answer: The speed at the bottom of the second ramp is 12.5 $m\,s^{-1}$ (to 3sf)

N17/41/Q3

63 A lorry of mass 7850 kg travels on a straight hill which is inclined at an angle of 3° to the horizontal. There is a constant resistance to motion of 1480 N.

(i) Find the power of the lorry's engine when the lorry is going up the hill at a constant speed of 10 $m\,s^{-1}$. [3]

(ii) Find the power of the lorry's engine at an instant when the lorry is going down the hill at a speed of 15 $m\,s^{-1}$ with an acceleration of 0.8 $m\,s^{-2}$. [3]

Answers: (i) 55 900 W (ii) 54 800 W

N17/43/Q2

64 A high-speed train of mass 490 000 kg is moving along a straight horizontal track at a constant speed of $85\ \mathrm{m\,s^{-1}}$. The engines are supplying 4080 kW of power.

(i) Show that the resistance force is 48 000 N. [1]

(ii) The train comes to a hill inclined at an angle $\theta°$ above the horizontal, where $\sin\theta° = \frac{1}{200}$. Given that the resistance force is unchanged, find the power required for the train to keep moving at the same constant speed of $85\ \mathrm{m\,s^{-1}}$. [3]

Answers: (i) 48 000 N (Answer given) (ii) 6.16 MW (to 3 sf) N18/41/Q2

65 A van of mass 2500 kg descends a hill of length 0.4 km inclined at 4° to the horizontal. There is a constant resistance to motion of 600 N and the speed of the van increases from $20\ \mathrm{m\,s^{-1}}$ to $30\ \mathrm{m\,s^{-1}}$ as it descends the hill. Find the work done by the van's engine as it descends the hill. [5]

Answer: 167 000 J N18/41/Q3

66 A particle of mass 1.2 kg moves in a straight line AB. It is projected with speed $7.5\ \mathrm{m\,s^{-1}}$ from A towards B and experiences a resistance force. The work done against this resistance force in moving from A to B is 25 J.

(i) Given that AB is horizontal, find the speed of the particle at B. [2]

(ii) It is given instead that AB is inclined at 30° below the horizontal and that the speed of the particle at B is $9\ \mathrm{m\,s^{-1}}$. The work done against the resistance force remains the same. Find the distance AB. [3]

Answer: $3.82\ \mathrm{ms^{-1}}$ *Answer*: 6.64 m N18/43/Q3

67 A van of mass 3200 kg travels along a horizontal road. The power of the van's engine is constant and equal to 36 kW, and there is a constant resistance to motion acting on the van.

(i) When the speed of the van is $20\ \mathrm{m\,s^{-1}}$, its acceleration is $0.2\ \mathrm{m\,s^{-2}}$. Find the resistance force. [3]

When the van is travelling at $30\ \mathrm{m\,s^{-1}}$, it begins to ascend a hill inclined at 1.5° to the horizontal. The power is increased and the resistance force is still equal to the value found in part **(i)**.

(ii) Find the power required to maintain this speed of $30\ \mathrm{m\,s^{-1}}$. [3]

(iii) The engine is now stopped, with the van still travelling at $30\ \mathrm{m\,s^{-1}}$, and the van decelerates to rest. Find the distance the van moves up the hill from the point at which the engine is stopped until it comes to rest. [4]

Answer: 1160 N *Answer*: 59900 W *Answer*: 721 m N18/43/Q6

68 A cyclist and his bicycle have a total mass of 90 kg. The cyclist starts to move with speed 3 m s^{-1} from the top of a straight hill, of length 500 m, which is inclined at an angle of $\sin^{-1} 0.05$ to the horizontal. The cyclist moves with constant acceleration until he reaches the bottom of the hill with speed 5 m s^{-1}. The cyclist generates 420 W of power while moving down the hill. The resistance to the motion of the cyclist and his bicycle, R N, and the cyclist's speed, $v\text{ m s}^{-1}$, both vary.

(i) Show that $R = \frac{420}{v} + 43.56$. [5]

(ii) Find the cyclist's speed at the mid-point of the hill. Hence find the decrease in the value of R when the cyclist moves from the top of the hill to the mid-point of the hill, and when the cyclist moves from the mid-point of the hill to the bottom of the hill. [3]

Answer: (ii) 4.12 ms^{-1} 38.1 17.9 N15/43/Q5

69 A straight hill AB has length 400 m with A at the top and B at the bottom and is inclined at an angle of 4° to the horizontal. A straight horizontal road BC has length 750 m. A car of mass 1250 kg has a speed of 5 m s^{-1} at A when starting to move down the hill. While moving down the hill the resistance to the motion of the car is 2000 N and the driving force is constant. The speed of the car on reaching B is 8 m s^{-1}.

(i) By using work and energy, find the driving force of the car. [5]

On reaching B the car moves along the road BC. The driving force is constant and twice that when the car was on the hill. The resistance to the motion of the car continues to be 2000 N. Find

(ii) the acceleration of the car while moving from B to C, [3]

(iii) the power of the car's engine as the car reaches C. [3]

Answer: (i) 1190 N (ii) 0.302 ms^{-2} (iii) 54100 W N15/43/Q7

70 A weightlifter performs an exercise in which he raises a mass of 200 kg from rest vertically through a distance of 0.7 m and holds it at that height.

(i) Find the work done by the weightlifter. [2]

(ii) Given that the time taken to raise the mass is 1.2 s, find the average power developed by the weightlifter. [2]

Answer: Work done by the weightlifter = 1400 J N15/41/Q1

Answer: Average Power developed by the weightlifter = 1170 W

71 A lorry of mass 24 000 kg is travelling up a hill which is inclined at 3° to the horizontal. The power developed by the lorry's engine is constant, and there is a constant resistance to motion of 3200 N.

(i) When the speed of the lorry is 25 m s^{-1}, its acceleration is 0.2 m s^{-2}. Find the power developed by the lorry's engine. [4]

(ii) Find the steady speed at which the lorry moves up the hill if the power is 500 kW and the resistance remains 3200 N. [2]

Answer: Power developed by the lorry's engine is 514 kW N15/41/Q3

Answer: Steady speed of the lorry up the hill is 31.7 ms^{-1}

MOMENTUM

We know that a force is needed to change the velocity of an object. The force required to change the object's velocity is dependent on the mass of the object. The precise relationship between force, mass and velocity can be established by combining Newton's second law with one of the equations of motion for constant acceleration.

Using Newton's second law, $F = ma$, and the equation of motion $v = u + at$ gives

$$F = m\left(\frac{v-u}{t}\right)$$

and hence

$$Ft = mv - mu$$

Note: The quantity Ft is called the 'impulse' of the force. For the purposes of the examination, knowledge and understanding of impulse is not required.

The right-hand side of the above equation is $mv - mu$, which is the change in the value of (mass × velocity). This product of the mass and the velocity of a body is called its **momentum**.

$$\text{momentum} = mv$$

Since momentum is a scalar multiple of velocity, which is a vector, then it follows that momentum is also a vector. The units used for momentum are those of Ft, that is, newton seconds (N s).

Note: You need to consider only motion in one dimension, that is, in a straight line.

EXAMPLE 1

A particle of mass 4 kg has an initial velocity of 5 m s^{-1}. Find

a) the momentum of the particle

b) the magnitude of the change in momentum if the velocity of the particle increases to 12 m s^{-1}.

EXAMPLE 2

A particle of mass 2.5 kg has an initial velocity of 3 m s^{-1}. Find the magnitude of the change in momentum of the particle if

a) the final speed of the particle is 5 m s^{-1} with the direction of motion unchanged

b) the final speed of the particle is 5 m s^{-1} with the direction of motion reversed.

COLLISIONS

When two bodies are in contact, according to Newton's third law they exert equal and opposite forces (F and $-F$) on each other. Whether the bodies are in contact for a long or a short time the time that each is in contact with the other (t) is the same. Hence, if no external forces act on the bodies the change in momentum of each must be equal and opposite. (Remember from Section 9.1 that $Ft = mv - mu$.)

Note: Two bodies in contact can be said to exert equal and opposite impulses on each other.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

This equation states that

total momentum before impact = total momentum after impact

This result is known as the **principle of conservation of linear momentum**. It applies for all collisions, provided no external force acts.

EXAMPLE 3

A particle of mass 2 kg travels at a speed of 5 m s^{-1}. It collides with a stationary particle of mass 8 kg. After the impact the 2 kg mass has a speed of 3 m s^{-1} and its direction of motion is reversed. Find the speed of the 8 kg mass after the impact.

EXAMPLE 4

Two particles, A and B, of masses 2 kg and 3 kg, respectively, are moving towards each other in the same straight line with speeds u m s^{-1} and $2u$ m s^{-1}, respectively. The two particles collide and coalesce. They continue to move in the initial direction of B before the impact, at a speed of 4 m s^{-1}. Calculate u.

Note: 'coalesce' means join together to form one object.

EXAMPLE 4

A lorry of mass 3 tonnes is moving at a constant speed of 4 m s^{-1} when it collides with another lorry, of mass 5 tonnes, moving at a speed of 2 m s^{-1} in the same direction. The speed of the 5-tonne truck is increased to 5 m s^{-1} by the collision. **Show that** the speed of the 3-tonne lorry is 1 m s^{-1} after the collision and find its direction of motion.

Summary exercise 9

1. Two particles, P and Q, are projected towards each other on a smooth horizontal surface. P has mass 0.6 kg and initial speed 2.8 m s^{-1}, and Q has mass 0.8 kg and initial speed 1.2 m s^{-1}. After a collision between P and Q the speed of P is 0.1 m s^{-1} and the direction of its motion is reversed.
   a) Calculate the change in momentum of P.
   b) Find the speed and direction of motion of Q after the collision.
2. Two particles of masses 50 g and 80 g are moving towards each other on a smooth horizontal surface. The initial speed of the 50 g mass is 3.9 m s^{-1} and that of the 80 g mass is 3.25 m s^{-1}. The particles collide and coalesce. Find the speed and direction of motion of the combined particle.

EXAM-STYLE QUESTIONS

3. Each of two wagons has an unloaded mass of 1500 kg. One of the wagons carries a load of mass m kg and the other wagon is unloaded. The wagons are moving towards each other on the same rails, each with a speed of 2 m s^{-1}, when they collide. Immediately after the collision the loaded wagon is at rest and the speed of the unloaded wagon is 3 m s^{-1}. Find the value of m.
4. A spacecraft of mass 40 000 kg docks with a space station of mass 160 000 kg. The spacecraft is travelling at 202 m s^{-1} immediately before docking takes place and the space station is travelling at 200 m s^{-1}. The docking is modelled by two particles moving in the same straight line that collide and coalesce.
   a) Calculate the exact speed of the spacecraft and space station after the docking.
   b) Calculate the total loss in kinetic energy during the docking.
5. Two spheres, P and Q, have masses 0.4 kg and 0.3 kg, respectively. The spheres are moving directly towards each other on a smooth horizontal surface and collide. Immediately before the collision, P has a speed of 6 m s^{-1} and Q has a speed of 4 m s^{-1}. Immediately after the collision the spheres move away from each other, P at a speed of v m s^{-1} and Q at a speed of $(5 - v)$ m s^{-1}. Find the value of v.

EXAM-STYLE QUESTION

6. A railway wagon A of mass 1200 kg, moving at a speed of 4 m s^{-1}, collides with a railway wagon B, which has mass 1800 kg and is moving towards A at a speed of 2 m s^{-1}. Immediately after the collision the speeds of A and B are equal.
   a) Given that the two wagons are moving in the same direction after the collision, find their common speed. **Determine** which wagon has changed its direction of motion.
   b) It is given instead that A and B are moving with equal speeds in opposite directions after the collision.
      i) Calculate the speed of the wagons after the collision.
      ii) Calculate the change in the momentum of A as a result of the collision.
7. A toy car of mass 240 g collides directly with a stationary toy lorry of mass 360 g. The car's speed is reduced by 3 m s^{-1}. Find the speed of the lorry after the collision.

EXAM-STYLE QUESTIONS

8. Two particles, of masses 0.2 kg and m kg, are moving towards each other and collide directly. Immediately before the collision the 0.2 kg particle has a speed of $4\,m\,s^{-1}$ and the m kg particle has a speed of $2\,m\,s^{-1}$.

a) Given that both particles are brought to rest by the collision, find the value of m.

b) Given instead that after the collision both particles move at a speed of $0.5\,m\,s^{-1}$, find all the possible values of m.

9. Two particles, A and B, of masses 2 kg and 1 kg, respectively, are initially moving towards each other on a smooth horizontal surface. Initially, the speed of A is $3\,m\,s^{-1}$ and the speed of B is $1\,m\,s^{-1}$. The particles collide. The direction of motion of A remains unchanged and the direction of motion of B is reversed. The loss of kinetic energy due to the collision is 5.25 J. Find the speeds of the particles after the collision.

10. Three smooth spheres, P, Q and R, of equal radii and of masses 5 kg, 4 kg and 6 kg, respectively, lie in that order in a straight line on a smooth horizontal plane. Initially, Q and R are at rest and P is moving towards Q at a speed of $9\,m\,s^{-1}$. After colliding with Q, sphere P continues to move in the same direction but at a speed of $2\,m\,s^{-1}$.

a) Find the speed of Q after this collision.

Sphere Q collides with sphere R. In this collision these two spheres coalesce to form an object S.

b) Find the speed of S after this collision.

c) Show that the total loss of kinetic energy in the system due to the two collisions is 131.25 J.

11. Two particles, A and B, are travelling in the same direction at different but constant speeds along a straight line when they collide. Particle A has mass 1.5 kg and speed $6\,m\,s^{-1}$. Particle B has mass 2.5 kg and speed $2\,m\,s^{-1}$. The particles coalesce during the collision. Find the speed of the combined particle after the collision.

12. Two model cars, A and B, have masses 250 grams and m grams, respectively. The cars move towards each other in a straight line on a horizontal table. They collide directly when the speed of A is $4\,m\,s^{-1}$ and the speed of B is $2\,m\,s^{-1}$. As a result of the collision the speed of A is reduced to $2\,m\,s^{-1}$ and it continues to move in the same direction as before the collision. The direction of B's motion is reversed and its speed immediately after the collision is $3\,m\,s^{-1}$. Find the value of m.

13. A particle P moves across a smooth horizontal surface in a straight line. P has mass 2 kg and speed $6\,m\,s^{-1}$. A particle Q, of mass 3 kg, is at rest on the surface. Particle P collides with particle Q.

a) Given that after the collision, P is at rest and Q moves away from P, find the speed of Q.

b) Given instead that after the collision, P and Q move away from each other with the same speed $v\,m\,s^{-1}$, find v.

14. Two particles, P and Q, have masses 1.2 kg and 0.4 kg, respectively. They are moving towards each other on a horizontal surface when they collide directly. Immediately before the collision the speed of P is $2.5\,m\,s^{-1}$ and the speed of Q is $1.5\,m\,s^{-1}$. Immediately after the collision, P and Q move in the same direction and the speed of Q is three times the speed of P.

a) Find the speed of P immediately after the collision.

b) Find the change in momentum of P.

c) Find the total loss in kinetic energy due to the collision.

15. Two railway coaches, A and B, of masses 800 kg and m kg, respectively, are moving in opposite directions towards each other when they collide. Immediately before the collision the speed of A is $5\,\mathrm{m\,s^{-1}}$ and the speed of B is $2.5\,\mathrm{m\,s^{-1}}$. Immediately after the collision the coaches join together and move with the same speed of $0.5\,\mathrm{m\,s^{-1}}$. The direction of motion of A is unchanged by the collision.

a) Find the value of m.

b) Find the change in momentum of B.

16. Particles A and B, with masses 0.5 kg and m kg, respectively, are moving on a smooth horizontal table in opposite directions and collide. Immediately before the collision the speed of A is $4\,\mathrm{m\,s^{-1}}$ and the speed of B is $2\,\mathrm{m\,s^{-1}}$. Due to the collision, A's direction of motion is reversed, and the momentum of A changes by 2.8 N s.

a) Find the speed of A immediately after the collision.

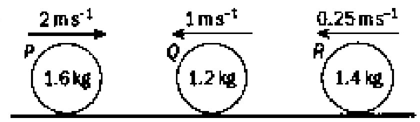
b) Given that the speed of B immediately after the collision is $1\,\mathrm{m\,s^{-1}}$, find the two possible values of m.

17. Two particles, P and Q, of masses $4m$ kg and m kg, respectively, are moving towards each other on a horizontal surface. Immediately before they collide, P has a speed of $2\,\mathrm{m\,s^{-1}}$ and Q has a speed of $11\,\mathrm{m\,s^{-1}}$. Immediately after the impact the direction of motion of both particles has been reversed and they are both travelling with the same speed $v\,\mathrm{m\,s^{-1}}$.

a) Show that $v = 1$.

The change in momentum of A during the collision is -18 N s.

b) Find the value of m

18. 

Three particles, P, Q and R, have masses 1.6 kg, 1.2 kg and 1.4 kg, respectively. The particles are moving in a straight line on a smooth horizontal table, with Q between P and R. The particle P is moving towards Q at a speed of $2\,\mathrm{m\,s^{-1}}$ and the particles Q and R are moving towards P at speeds of $1\,\mathrm{m\,s^{-1}}$ and $0.25\,\mathrm{m\,s^{-1}}$, respectively.

a) P collides with Q. As a result of this collision the direction of motion of Q is reversed and its speed remains $1\,\mathrm{m\,s^{-1}}$. Find the speed of P after the collision.

b) Q collides with R.

i) Find the total momentum of Q and R in the direction of Q's motion immediately before the collision takes place, and verify that the direction of motion of R is reversed as a result of this collision.

ii) Given that Q is brought to rest by this collision, find the speed of R immediately after this collision.

19. Two particles, A and B, have masses 0.6 kg and 0.2 kg, respectively. A and B are simultaneously projected towards each other in the same straight line on a horizontal surface at speeds of $5\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$, respectively. Before A and B collide the only horizontal force acting is friction, and each particle decelerates at $0.5\,\mathrm{m\,s^{-2}}$. The particles collide and coalesce 3 s after projection.

a) Find the speed of each particle immediately before the collision.

b) Find the speed of the combined particle immediately after the collision.

20. AB is a line of greatest slope, of length 6 m, on a smooth plane inclined at 30° to the horizontal. Particles P and Q, of masses 0.2 kg and 0.5 kg, respectively, move along AB, with P below Q. The particles are moving upwards, P at a speed of 8 m s^{-1} and Q at a speed of 2 m s^{-1}, when they collide at a point 3 m from B. Particle P is instantaneously at rest after the collision.

a) Determine whether Q reaches B in the subsequent motion.

b) Find the time between the collision and P's arrival at A.

Summary exercise 9 page 120

1. **a)** 1.74 N s
   b) 0.975 m s^{-1}; direction reverses
2. 0.5 m s^{-1}, in the direction of the 80 g mass before the collision
3. 2250
4. **a)** 200.4 m s^{-1} **b)** 64 000 J
5. 0.429 (3 s.f.)
6. **a)** 0.727 m s^{-1} (3 s.f.); B changes direction.
   b) **i)** 8 m s^{-1} **ii)** 18 000 N s
7. 2 m s^{-1}
8. **a)** 0.4 **b)** 0.28, 0.6, 0.36
9. A: 1.5 m s^{-1}, B: 2 m s^{-1}
10. **a)** 8.75 m s^{-1} **b)** 3.5 m s^{-1}
11. 3.5 m s^{-1}
12. 100
13. **a)** 4 m s^{-1} **b)** 12
14. **a)** 1 m s^{-1} **b)** −1.8 N s **c)** 1.8 J
15. **a)** 1200 **b)** 3600 N s
16. **a)** 1.6 m s^{-1} **b)** 0.933 or 2.8
17. **a)** Proof **b)** 1.5
18. **a)** 0.5 m s^{-1}
    b) **i)** 0.85 N s; proof **ii)** 0.607 m s^{-1}
19. **a)** A: 3.5 m s^{-1}; B: 0.5 m s^{-1}
    b) 2.5 m s^{-1}
20. **a)** Q does not reach B. **b)** 1.10 s

HOMEWORK: MOMENTUM PRACTICE QUESTIONS

1 Particle P has mass m kg and particle Q has mass $3m$ kg. The particles are moving in opposite directions along a smooth horizontal plane when they collide directly. Immediately before the collision P has speed $4u$ ms^{-1} and Q has speed ku ms^{-1}, where k is a constant. As a result of the collision the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of k.

(4)

Ans : k = $\frac{4}{3}$ M1 Dynamics Q1

2 Two particles A and B have mass 0.4 kg and 0.3 kg respectively. They are moving in opposite directions on a smooth horizontal table and collide directly. Immediately before the collision, the speed of A is 6 m s^{-1} and the speed of B is 2 m s^{-1}. As a result of the collision, the direction of motion of B is reversed and its speed immediately after the collision is 3 m s^{-1}. Find

(a) the speed of A immediately after the collision, stating clearly whether the direction of motion of A is changed by the collision,

(4)

Ans : v = (+) 2.25 ms^{-1} M1 Dynamics Q3

3 (a) Two particles A and B, of mass 3 kg and 2 kg respectively, are moving in the same direction on a smooth horizontal table when they collide directly. Immediately before the collision, the speed of A is 4 m s^{-1} and the speed of B is 1.5 m s^{-1}. In the collision, the particles join to form a single particle C.

Find the speed of C immediately after the collision.

(3)

(b) Two particles P and Q have mass 3 kg and m kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table. Each particle has speed 4 m s^{-1}, when they collide directly. In this collision, the direction of motion of each particle is reversed. The speed of P immediately after the collision is 2 m s^{-1} and the speed of Q is 1 m s^{-1}. Find

(i) the value of m,

(3)

Ans : a) v = 3 ms^{-1} b) m = 3.6 M1 Dynamics Q4

4 Two small steel balls A and B have mass 0.6 kg and 0.2 kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table when they collide directly. Immediately before the collision, the speed of A is 8 m s^{-1} and the speed of B is 2 m s^{-1}. Immediately after the collision, the direction of motion of A is unchanged and the speed of B is twice the speed of A. Find

(a) the speed of A immediately after the collision,

(5)

Ans : v = 4.4 ms^{-1} M1 Dynamics Q5

5 A particle P of mass 1.5 kg is moving along a straight horizontal line with speed 3 m s^{-1}. Another particle Q of mass 2.5 kg is moving, in the opposite direction, along the same straight line with speed 4 m s^{-1}. The particles collide. Immediately after the collision the direction of motion of P is reversed and its speed is 2.5 m s^{-1}.

(a) Calculate the speed of Q immediately after the impact.

(3)

(b) State whether or not the direction of motion of Q is changed by the collision.

(1)

Ans : a) v = - 0.7 ms^{-1} so speed = 0.7 ms^{-1} b) Direction Q unchanged M1 Dynamics Q6

6 A particle P of mass 2 kg is moving with speed u m s^{-1} in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass 4 kg which is at rest on the same horizontal plane. Immediately after the collision, P and Q are moving in opposite directions and the speed of P is one-third the speed of Q.

(a) Show that the speed of P immediately after the collision is $\frac{1}{5}u$ m s^{-1}.

(4)

M1 Dynamics Q9

7 Two trucks A and B, moving in opposite directions on the same horizontal railway track, collide. The mass of A is 600 kg. The mass of B is m kg. Immediately before the collision, the speed of A is 4 m s^{-1} and the speed of B is 2 m s^{-1}. Immediately after the collision, the trucks are joined together and move with the same speed 0.5 m s^{-1}. The direction of motion of A is unchanged by the collision. Find

(a) the value of m,

(4)

Ans : m = 840 kg M1 Dynamics Q10

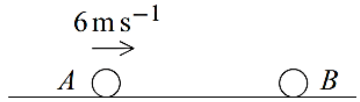
8 A railway truck P of mass 2000 kg is moving along a straight horizontal track with speed 10 m s^{-1}. The truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision Q moves with speed 5 m s^{-1}. Calculate

(a) the speed of P immediately after the collision,

(3)

Ans : v = 2.5 ms^{-1} M1 Dynamics Q13

9 A particle A moves across a smooth horizontal surface in a straight line. The particle A has mass 2 kg and speed $6\ \text{m s}^{-1}$. A particle B, which has mass 3 kg, is at rest on the surface. The particle A collides with the particle B.



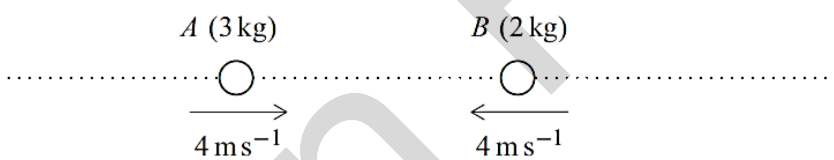
(a) If, after the collision, A is at rest and B moves away from A, find the speed of B. *(3 marks)*

(b) If, after the collision, A and B move away from each other with speeds $v\ \text{m s}^{-1}$ and $4v\ \text{m s}^{-1}$ respectively, as shown in the diagram below, find the value of v.

(3 marks)

Ans : a) v = 4 ms^{-1} b) v = 1.2 ms^{-1} M1 Momentum 2 Q1

10 Two particles A and B have masses of 3 kg and 2 kg respectively. They are moving along a straight horizontal line towards each other. Each particle is moving with a speed of $4\ \text{m s}^{-1}$ when they collide.

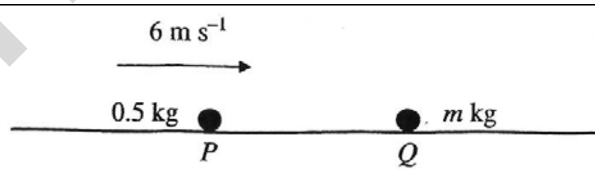


(a) If the particles coalesce during the collision to form a single particle, find the speed of the combined particle after the collision. *(3 marks)*

(b) If, after the collision, A moves in the same direction as before the collision with speed $0.4\ \text{m s}^{-1}$, find the speed of B after the collision. *(3 marks)*

Ans : v = 0.8 b) v = 1.4 M1 Momentum 2 Q1

11



A particle P of mass 0.5 kg is travelling with speed $6\ \text{m s}^{-1}$ on a smooth horizontal plane towards a stationary particle Q of mass m kg (see diagram). The particles collide, and immediately after the collision P has speed $0.8\ \text{m s}^{-1}$ and Q has speed $4\ \text{m s}^{-1}$.

(i) Given that both particles are moving in the same direction after the collision, calculate m. **[3]**

(ii) Given instead that the particles are moving in opposite directions after the collision, calculate m. **[3]**

Answer: i) 0.65 ii) 0.85 M1 Momentum Q1

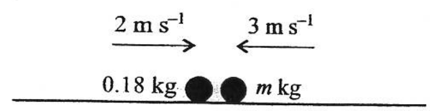
12 Each of two wagons has an unloaded mass of 1200 kg. One of the wagons carries a load of mass m kg and the other wagon is unloaded. The wagons are moving towards each other on the same rails, each with speed $3\ \mathrm{m\,s^{-1}}$, when they collide. Immediately after the collision the loaded wagon is at rest and the speed of the unloaded wagon is $5\ \mathrm{m\,s^{-1}}$. Find the value of m. [5]

Answer: 2000 M1 momentum Q1

13 An ice skater of mass 40 kg is moving in a straight line with speed $4\ \mathrm{m\,s^{-1}}$ when she collides with a skater of mass 60 kg moving in the opposite direction along the same straight line with speed $3\ \mathrm{m\,s^{-1}}$. After the collision the skaters move together with a common speed in the same straight line. Calculate their common speed, and state their direction of motion. [5]

Answer: i) 0.2, same as heavier skater M1 Momentum Q2

14



Two particles of masses 0.18 kg and m kg move on a smooth horizontal plane. They are moving towards each other in the same straight line when they collide. Immediately before the impact the speeds of the particles are $2\ \mathrm{m\,s^{-1}}$ and $3\ \mathrm{m\,s^{-1}}$ respectively (see diagram).

(i) Given that the particles are brought to rest by the impact, find m. [3]

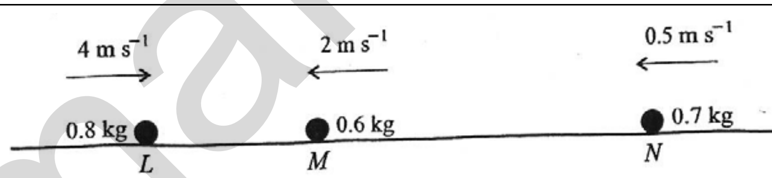
(ii) Given instead that the particles move with equal speeds of $1.5\ \mathrm{m\,s^{-1}}$ after the impact, find

(a) the value of m, assuming that the particles move in opposite directions after the impact, [3]

(b) the two possible values of m, assuming that the particles coalesce. [4]

Answer: i) 0.12 ii) a) 0.14 b) 0.42, 0.02 M1 Momentum Q4

15



Three uniform spheres L, M and N have masses 0.8 kg, 0.6 kg and 0.7 kg respectively. The spheres are moving in a straight line on a smooth horizontal table, with M between L and N. The sphere L is moving towards M with speed $4\ \mathrm{m\,s^{-1}}$ and the spheres M and N are moving towards L with speeds $2\ \mathrm{m\,s^{-1}}$ and $0.5\ \mathrm{m\,s^{-1}}$ respectively (see diagram).

(i) L collides with M. As a result of this collision the direction of motion of M is reversed, and its speed remains $2\ \mathrm{m\,s^{-1}}$. Find the speed of L after the collision. [4]

(ii) M then collides with N.

(a) Find the total momentum of M and N in the direction of M's motion before this collision takes place, and deduce that the direction of motion of N is reversed as a result of this collision. [4]

(b) Given that M is at rest immediately after this collision, find the speed of N immediately after this collision. [2]

16 **(i)**

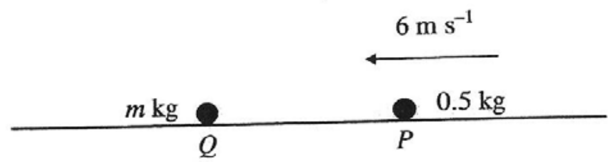


Fig. 1

A particle P of mass 0.5 kg is projected with speed 6 m s^{-1} on a smooth horizontal surface towards a stationary particle Q of mass m kg (see Fig. 1). After the particles collide, P has speed v m s^{-1} in its original direction of motion, and Q has speed 1 m s^{-1} more than P. Show that $v(m + 0.5) = -m + 3$. **[3]**

(ii)

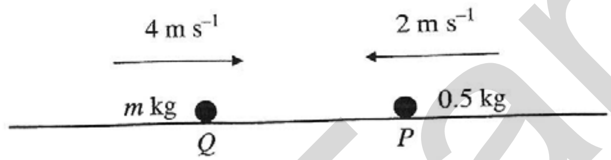


Fig. 2

Q and P are now projected towards each other with speeds 4 m s^{-1} and 2 m s^{-1} respectively (see Fig. 2). Immediately after the collision the speed of Q is v m s^{-1} with its direction of motion unchanged and P has speed 1 m s^{-1} more than Q. Find another relationship between m and v in the form $v(m + 0.5) = am + b$, where a and b are constants. **[4]**

(iii) By solving these two simultaneous equations show that $m = 0.9$, and hence find v. **[4]**

Answer: ii) v(m+0.5)=4m-1.5 iii) 1.5 M1 Momentum Q5

17 A railway wagon A of mass 2400 kg and moving with speed 5 m s^{-1} collides with railway wagon B which has mass 3600 kg and is moving towards A with speed 3 m s^{-1}. Immediately after the collision the speeds of A and B are equal.

(i) Given that the two wagons are moving in the same direction after the collision, find their common speed. State which wagon has changed its direction of motion. [5]

(ii) Given instead that A and B are moving with equal speeds in opposite directions after the collision, calculate

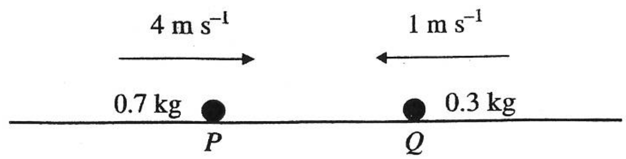
(a) the speed of the wagons,

(b) the change in the momentum of A as a result of the collision.

[5]

Answer: i) 0.2 ii) a) 1 b) 14400 M1 Momentum Q5

18



Two particles P and Q have masses 0.7 kg and 0.3 kg respectively. P and Q are simultaneously projected towards each other in the same straight line on a horizontal surface with initial speeds of $4\ \text{m s}^{-1}$ and $1\ \text{m s}^{-1}$ respectively (see diagram). Before P and Q collide the only horizontal force acting on each particle is friction and each particle decelerates at $0.4\ \text{m s}^{-2}$. The particles coalesce when they collide.

(i) Given that P and Q collide 2 s after projection, calculate the speed of each particle immediately before the collision, and the speed of the combined particle immediately after the collision. **[6]**

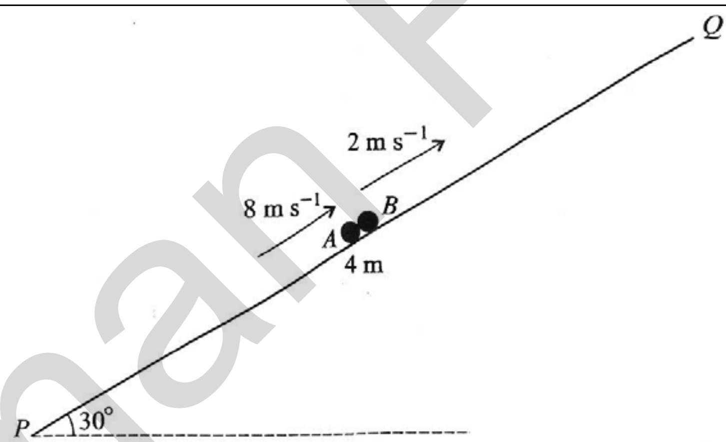
(ii) Given instead that P and Q collide 3 s after projection,

(a) sketch on a single diagram the (t, v) graphs for the two particles in the interval $0 \leqslant t < 3$, **[3]**

(b) calculate the distance between the two particles at the instant when they are projected. **[6]**

Answer: i) P=3.2 Q=0.2 v=2.18 ii) b) 11.45 m M1 momentum Q7

19



PQ is a line of greatest slope, of length 4 m, on a smooth plane inclined at 30° to the horizontal. Particles A and B, of masses 0.15 kg and 0.5 kg respectively, move along PQ with A below B. The particles are both moving upwards, A with speed $8\ \text{m s}^{-1}$ and B with speed $2\ \text{m s}^{-1}$, when they collide at the mid-point of PQ (see diagram). Particle A is instantaneously at rest immediately after the collision.

(i) Show that B does not reach Q in the subsequent motion. [8]

(ii) Find the time interval between the instant of A's arrival at P and the instant of B's arrival at P. [6]

Answer: ii) 1.27 seconds (with g=9.8) M1 Momentum q7

4 Mechanics (for Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$$\sin(90^\circ - \theta) \equiv \cos\theta,\ \cos(90^\circ - \theta) \equiv \sin\theta,\ \tan\theta \equiv \frac{\sin\theta}{\cos\theta},\ \sin^2\theta + \cos^2\theta \equiv 1.$$

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a 'particle'. Examination questions may involve extended bodies in a 'realistic' context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

4.1 Forces and equilibrium

Candidates should be able to:	Notes and examples
• identify the forces acting in a given situation	e.g. by drawing a force diagram.
• understand the vector nature of force, and find and use components and resultants	Calculations are always required, not approximate solutions by scale drawing.
• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero	Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	
• use the model of a 'smooth' contact, and understand the limitations of this model	
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leqslant \mu R$, as appropriate	Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.
• use Newton's third law.	e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.

4 Mechanics

4.2 Kinematics of motion in a straight line

Candidates should be able to:	Notes and examples
• understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities	Restricted to motion in one dimension only. The term 'deceleration' may sometimes be used in the context of decreasing speed.
• sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that – the area under a velocity–time graph represents displacement, – the gradient of a displacement–time graph represents velocity, – the gradient of a velocity–time graph represents acceleration	
• use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration	Calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.
• use appropriate formulae for motion with constant acceleration in a straight line.	Questions may involve setting up more than one equation, using information about the motion of different particles.

4.3 Momentum

Candidates should be able to:	Notes and examples
• use the definition of linear momentum and show understanding of its vector nature	For motion in one dimension only.
• use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies.	Including direct impact of two bodies where the bodies coalesce on impact. Knowledge of impulse and the coefficient of restitution is not required.

4 Mechanics

4.4 Newton's laws of motion

Candidates should be able to:	Notes and examples
• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod	If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.
• use the relationship between mass and weight	$W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value 10 (m s^{-2}) for g is expected.
• solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration	Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.
• solve simple problems which may be modelled as the motion of connected particles.	e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar.

4.5 Energy, work and power

Candidates should be able to:	Notes and examples
• understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force	$W = Fd\cos\theta$; Use of the scalar product is not required.
• understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae	
• understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy	Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.
• use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion	Including calculation of (average) power as $\dfrac{\text{Work done}}{\text{Time taken}}$. $P = Fv$.
• solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.	

5 List of formulae and statistical tables (MF19)

PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone or pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

Area of curved surface of cone = $\pi r \times \text{slant height}$

Arc length of circle $= r\theta$ (θ in radians)

Area of sector of circle $= \frac{1}{2}r^2\theta$ (θ in radians)

Algebra

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d\,, \qquad S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}\,, \qquad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)\,, \qquad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial series:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n, \text{ where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots, \text{ where } n \text{ is rational and } |x| < 1$$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1, \qquad 1 + \tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1} x \leqslant \tfrac{1}{2}\pi, \qquad 0 \leqslant \cos^{-1} x \leqslant \pi, \qquad -\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Differentiation

f(*x*)	**f′(*x*)**
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$v\frac{du}{dx} + u\frac{dv}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $x = f(t)$ and $y = g(t)$ then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$	
x^n	$\dfrac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	
e^x	e^x	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\sec^2 x$	$\tan x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert$	$(\lvert x\rvert < a)$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$$

MECHANICS

Uniformly accelerated motion

$$v = u + at, \qquad s = \tfrac{1}{2}(u+v)t, \qquad s = ut + \tfrac{1}{2}at^2, \qquad v^2 = u^2 + 2as$$

FURTHER MECHANICS

Motion of a projectile

Equation of trajectory is:

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Elastic strings and springs

$$T = \frac{\lambda x}{l}, \qquad E = \frac{\lambda x^2}{2l}$$

Motion in a circle

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \quad \text{or} \quad \frac{v^2}{r}$$

Centres of mass of uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere of radius r: $\frac{3}{8}r$ from centre

Hemispherical shell of radius r: $\frac{1}{2}r$ from centre

Circular arc of radius r and angle 2α: $\dfrac{r\sin\alpha}{\alpha}$ from centre

Circular sector of radius r and angle 2α: $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\frac{3}{4}h$ from vertex